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ABSTRACT

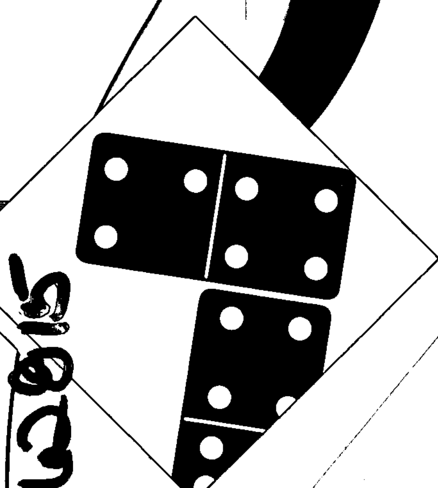
This book was developed for teachers, youth group leaders, after-school child care providers, and parents, who may not have the time or the expertise to develop strategies for preparing students to be effective problem solvers. The content is organized in a pyramid style to make it easy to locate and grasp the information provided. Information on effective strategies for teaching general real-life problem solving is provided first. Similar information specific to real-life math problem solving follows. Together these two sections lay a foundation to prepare teachers to successfully deliver the learning activities subsequently provided. The Learning Activities section is organized by strand as identified by the Ohio Mathematics Proficiency Outcomes. Each section begins with an index of the activities included in that strand. Appendices provide additional details that can be used to facilitate the development of more complex skills. Appendix A contains references; Appendix B lists resources, i.e., books, software, Internet sites, and materials suppliers; Appendix C features common math vocabulary; Appendix D contains the Ohio 4th-, 6th-, and 9th-grade Mathematics Proficiency Outcomes; and Appendix E includes matrices of learning activities, proficiency outcomes, and process skills designed to help focus learning activities on specific mathematical and process skills. (ASK)

Based on National Mathematics Standards

ED 434 009

Fun with Math

Real-Life Problem Solving for Grades 4-8



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An Important Note to Non-Math Teachers and Others Who Are Considering Using This Book

You don't have to be a certified math teacher to use this book effectively. *Fun with Math* was developed for teachers, youth group leaders, after-school child care providers, and parents who may not have the time or expertise for preparing students to be effective problem solvers.

In fact, many of the teachers who developed the lessons in this book are not math specialists. Like many elementary school teachers, they did not have extensive training in mathematics education. Early in their teaching careers, most of them didn't conduct much hands-on math in their classrooms because they lacked confidence in their abilities, believed they didn't have adequate educational background, and/or didn't have adequate time to develop innovative instructional activities.

Despite their beliefs and concerns, these teacher-writers decided to *try* a simple, hands-on activity with their students. The result—**they realized that when they taught math in an inquiry-based, hands-on manner, students learned, understood, and remembered more**. These teachers and many others, who began teaching through inquiry, found that they could teach math in a way that works for them *and* for their students.

Thus, from their own experiences in developing math teaching skills and strategies, the teachers who contributed to this book understand that readers who use it may experience similar feelings of inadequate preparation, low confidence, and/or lack of preparation time. That's why these teacher-writers designed the lessons in *Fun with Math*—so that any teacher, youth group leader, after-school child care provider, or parent can successfully provide hands-on math instruction.

Each teacher-tested lesson provides clear, step-by-step instructions and plenty of background resources. In addition, the hands-on, trial-and-error nature of the activities will allow your students to work through the lessons with you and retain the learning longer. And if mistakes are made in the activities, learning has still taken place; students will know what to avoid the next time. **So, pick a learning activity that you would like to try and begin!**

Based on National Mathematics Standards

Fun with Math

Real–Life Problem Solving for Grades 4-8

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In-Service Training

For training in the use of *Fun with Math*, contact the Ohio State University's Vocational Instructional Materials Laboratory (VIML) at 800/848–4815, ext. 2–8300. Training can be provided at your school or at the VIML.

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Project Overview

Both *Fun with Math: Real-Life Problem Solving for Grades K–3* and *Fun with Math: Real-Life Problem Solving for Grades 4–8* were written and piloted by a wide range of teachers and teacher trainers. These books were developed as resources for teachers to use in helping students improve their problem-solving skills in the context of real-life settings and situations.

The development of books was funded by an Urban/Rural Opportunities Grant through the Buckeye Hills Collaborative. Both books are distributed by The Ohio State University's Vocational Instructional Materials Laboratory (VIML), which is part of the Center on Education and Training for Employment. Similar books for grades 9–12 and adults have also been developed and are distributed by the VIML. The sales office can be reached by calling 800/848–4815 or faxing 614/292–1260.

In addition, the VIML's professional staff provides coaching and workshops for teachers and trainers that prepares them to effectively use these and other math- and science-related materials. The VIML staff also does training in problem solving with math and science for grades 9–12 and adults, using the ACT Work Keys System™. For further information, contact the VIML directly at 800/848–4815, ext. 2–8300 or 614/292–8300.

Acknowledgments

Many people have committed their time and talents to help make this book a useful resource for teachers and others wishing to give students opportunities to solve real-life math problems.

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* Designates members of the Buckeye Hills Collaborative Partnership. All pilot teachers were members of the Partnership.

Contents

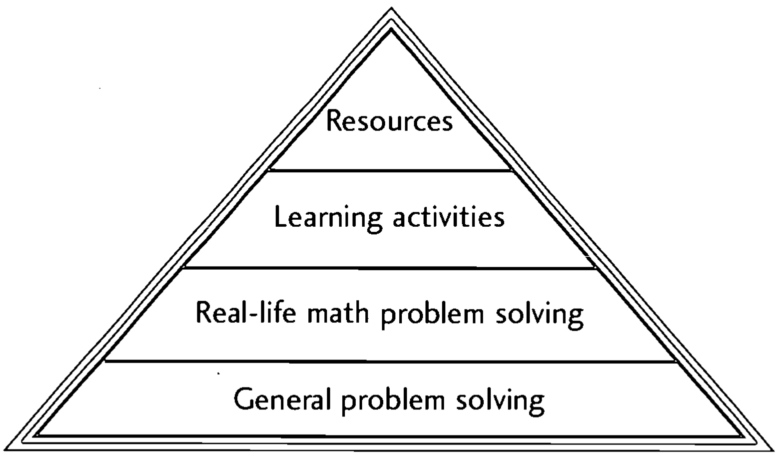
Appendices

About This Book

Fun with Math: Real-Life Problem Solving was developed for teachers who may not have the time or the expertise to develop strategies for preparing students to be effective problem solvers. Thus, significant attention has been given to making it **user-friendly**. Both the organization of the book's content and the content development were carefully planned to achieve this outcome.

Content organization

The book's content is organized in a pyramid style to make it easy to locate and grasp the information provided. Information about effective strategies for teaching *general* real-life problem solving are provided first. Similar information specific to real-life *math* problem solving follows. Together, these two sections lay a foundation to prepare teachers to successfully deliver the learning activities provided in *Fun with Math.*



The Learning Activities section is organized by strand, as identified by the Ohio Mathematics Proficiency Outcomes. Each section begins with an index of the activities included in that strand.

Fun with Math's appendices provide additional details that can be used to facilitate the development of more complex skills.

Appendix A—References
Appendix B—Resources (i.e., books, software, Internet sites, and materials suppliers)
Appendix C—Common math vocabulary
Appendix D—Ohio 4th-, 6th-, and 9th-grade Mathematics Proficiency Outcomes
Appendix E—Matrices of learning activities, proficiency outcomes, and process skills, designed to help you focus learning activities on specific mathematical and process skills.

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Content development

The *Fun with Math* development team followed the guidelines listed below in creating this book:

Hint: If you missed the note in the inside front cover, go ahead and read it now. It provides insights and encouragement from teachers who once felt overwhelmed about teaching real-life problem solving.

- *Fun with Math* was written for **non-math teachers**. You don't need formal training in math to make full use of this book. However, even formally trained math teachers will find the book's practical focus on applications to real-life situations valuable.

- *Fun with Math* **supports what teachers are already doing** by offering ideas and materials for achieving their current goals and objectives. It is not a new program or a new curriculum—it is a resource that **compliments** existing instructional efforts.

- *Fun with Math* is not intended to be a complete curriculum. Instead, it **supplements the curriculum** by giving teachers a wide variety of learning activities to use as needed.

- *Fun with Math* was developed with consideration for the **National Mathematics Standards** and the Benchmarks for Science Literacy. The book's goal is to help students become more effective real-life problem solvers. The learning activities were designed within the following philosophy, which reflects the key principles underlying the national mathematics standards. These were also validated by the TIMSS findings.

  - ✓ **All students can learn**, regardless of differing abilities, economic status, learning styles, and academic strengths.

  - ✓ **Learning is an active process.**

  - ✓ **Math and science education should model the process by which explorations are conducted in the real world.**

- The specific **Ohio Mathematics Proficiency Outcomes** addressed by each learning activity in *Fun with Math* are noted on the first page of each activity. Proficiency Outcomes are listed in Appendix D, pp. 422–427. These outcomes are measured by proficiency tests in Ohio; other states identify and assess similar competencies.

With this overview of the book in mind, look at your present needs. What goals do you hope this book will help you achieve? List these if you wish. The next section (*Where Do I Begin?*) will help you get started on these goals.

Where Do I Begin?

So here you are, interested in using *Fun with Math* as a resource for helping students develop and/or practice problem-solving skills and wondering where to begin. Everyone reading this book has a unique blend of experience, ideas, resources, and needs. Therefore, uses for the book will vary. However, pilot teachers provided a step-by-step model to help you get started. Once you gain experience, you can customize the model for your unique situation.

Steps for getting started

Read *Effective Instructional Methods for Real-Life Problem-Solving and Effective Instructional Strategies for Real-Life Math Problem-Solving* (pp. 5–40) for background information about real-life problem solving. These sections will help you **understand the principles** involved in teaching real-life problem solving. You may wish to highlight some principles, models, or ideas to refer back to when implementing learning activities from this book.

1. **Select a strand** to focus on that matches your instructional needs.

2. **Review the learning activities** in that strand.

3. **Choose an activity** that you'd like to try.

4. Check to be sure that students have the **prerequisite skills** needed to complete this activity. If they don't, provide one or more lessons to help students build the knowledge and/or skills needed to complete the learning activity. Many *Fun with Math* lessons provide engagement activities that can be used for such preparation.

5. **Gather the materials** needed for the learning activity.

6. **Review the real-life applications** and add others from your knowledge of students and your local community (e.g., businesses, social factors).

7. Organize a **lesson plan** that includes these components:

    - Provide opportunities for students to **solve problems cooperatively** (e.g., to work individually but share ideas with classmates, to work in assigned pairs, and to work in assigned small groups.)

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- Vary student grouping. Have students work on some tasks individually, some in pairs, and some in small groups of 3–5 students. (Refer to pp. 21–23 for ideas about cooperative/collaborative learning and pp. 24–27 for ideas about instructional equity.)

- Have students keep a **journal** in which they reflect upon what they learn and the processes used in solving problems.

8. **Assess student learning** in a variety of ways. Suggestions are provided in the Evaluation section of each learning activity.

9. **Supplement *Fun with Math* materials** with other math resources as needed. Suggestions are provided in the Resources for Teachers section of each learning activity. (Resources are also listed in Appendix B, pp. 408–417.)

10. Plan **additional instructional strategies** for students whose skills did not improve as much as necessary. The extension activities in each lesson can be used for this purpose, as well as for reinforcement.

Effective Strategies for Teaching Real-Life Problem Solving

Teaching Through Inquiry

Rationale for teaching through inquiry

Students come to school with a wealth of knowledge. Not all students have the same knowledge, but they all possess knowledge. The teacher's challenge is to identify what knowledge each student possesses about any given subject or topic and utilize that knowledge as the foundation for building greater understanding. Teachers must become skilled questioners to help students draw upon the knowledge they already have so they may *discover* and *develop* other, related concepts. When students engage in mathematical or other explorations, they use their prior knowledge, manipulatives (or models), logical reasoning, trial and error, and many other processes to construct new knowledge. When students are required to construct their own knowledge from an inquiry-based exploration, the new knowledge has greater meaning and value. Therefore the student retains the new knowledge longer and applies it to new situations more efficiently and effectively. Especially in the areas of math, science, and technology, students find the process of discovering concepts fun and empowering. And an empowered student delights in exploring new applications and concepts, further strengthening their understanding of the subject matter.

About inquiry-based learning

When using inquiry-based learning, teachers provide students with opportunities to ask questions, then answer those questions for themselves through investigations.[1] Inquiry-based instruction includes these characteristics:[2]

- Starts with one or more problems or open-ended questions.
- Engages students actively.
- Helps students make real-world connections and applications.
- Concentrates on students' collection and use of evidence.

[1] *Science With Reason* (1995) p. 7.

[2] *National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School Mathematics* (1989) and *Benchmarks for Science Literacy* (1993) professional development CD-ROM.

- Requires clear expression (e.g., written and verbal communication).
- Often uses a team approach.
- Does not separate knowing from finding out.
- Welcomes curiosity and rewards creativity, encouraging students to make mistakes and to learn from them.
- De-emphasizes memorizing facts and technical vocabulary.
- Provides opportunities for students to have successes, which increases their self-confidence.

Strategy for developing real-life, open-ended questions

Teachers can change the questions they ask so there are *many* possible correct answers instead of just *one*. A traditional math problem such as, "What percentage of the chairs in the room are blue?" has one correct answer. In contrast, a problem that states, "Find examples of objects in your life that make up 20% of a whole" has many answers. These open-ended math problems become personalized to students' lives by encouraging them to create solutions from their own experiences.

Five Es of Instruction: A model for structuring inquiry-based learning

Many teachers use an inquiry-based instructional model called the **Five Es of Instruction** (the Five Es) with great success. The Five Es is an expansion of the learning cycle. It can be used as a *format for structuring lessons* that allows students to learn through an inquiry-based process. The Five Es model includes the following components:

Engagement *How will you start the class and get students interested?* What "hook" will motivate your students to learn more about the topic at hand? Engagement usually involves students in a thought-provoking or discrepant event. It may include a demonstration and usually involves an all-class or small-group discussion. Often in math instruction, the engagement step helps students identify their preconceived ideas about a given concept. Note: This step is optional. Sometimes, "hooks" are integrated into the exploration activities.

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Exploration *What investigations, discussions, individual activities, and/or cooperative group activities will be used?* In other words, what hands-on activities will guide students to make observations and collect data? Note: In this stage, students are not given explanations about what to expect or why things happen the way they do, and no new vocabulary is introduced.

Explanation *How will you help students analyze what they have experienced?* In this step, your job is to help students draw conclusions and form new ideas from their observations and the patterns that surfaced during exploration. Traditional instructional strategies (e.g., demonstrations, lecture-discussions, textbook assignments, group reports, videotapes, and library research) are helpful strategies for completing this step.

Evaluation *How are you going to assess student learning?* In other words, how will you know that students understand the concepts and processes being taught?

Extensions *What possibilities exist to build upon the learning accomplished by this activity?* In this step, you will help students use what they learned to solve new problems. In addition, students can complete some of the extension activities at home. (See the Home Connections sections in the learning activities.)

The Five Es model was used to structure the *Fun with Math* learning activities. In many cases, the explanation step is included as a class discussion at the end of the exploration.

The teacher's role in inquiry-based learning

Using inquiry-based instructional strategies takes teachers out of the traditional role of "bestower of knowledge" and puts them into the role of a learning **facilitator**, in which they *coordinate the learning experience* by:

1. Determining the theme or topic that will be studied.

2. Organizing exploratory experiences for students.

3. Facilitating questioning.

4. Providing a framework for collecting information and making generalizations.

5. Giving students facts (to supplement what students have discovered on their own).

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Thus, inquiry-based instruction puts students in the center of the learning process where they can lead their own learning and moves teachers to the sidelines where they can facilitate (guide) and provide resources.

A facilitating teacher uses the Five Es (or another inquiry-based learning format) to provide a motivating (engagement) activity, explains what is expected in the exploration activity, and lets students discover answers for themselves. Once exploration has taken place, the facilitator helps students synthesize what they have discovered, label it, and apply that understanding to other situations. A facilitator also provides resources—from materials to ideas, referrals, and questions—that help students pursue and capture learning on their own. When giving ideas and referrals or answering questions, it is important to give students only minimal information (i.e., as little as you can while still giving them enough to keep the discovery process moving) and let them discover the rest for themselves.

In summary, **teachers facilitate the learning process rather than leading it.** To be successful in the facilitator role, you will likely need to develop some new thought patterns and skills, such as:

- **Allow students to be at the center of the learning process**. In planning and implementing lessons, make a conscious effort to take on a facilitative role in the learning process. The learning activities in *Fun with Math* have been designed with students at the center of the learning process; use them as examples to help you develop student-centered lesson plans.

- **Structure the learning experience so students can learn and practice problem-solving skills**. Hands-on exploration in teams is a very effective instructional strategy when focusing on problem-solving skills. Again, this book provides many examples of practical problem-solving activities.

- **Help students attain their goals without telling them the answers** by staying in the resource person role. Questioning techniques are especially helpful in this role. See the bullet "Use open-ended questioning techniques" for more information.

- **Lead discussions** by asking students to share their hypotheses, predictions, and test results. This discussion method encourages students to synthesize their learning and communicate that knowledge to peers. Students often need help *from the sidelines* as they learn these skills. Questions that teachers might use to enable students to express their ideas and reactions include:[3]

[3] This list was adapted from Matthew Lipman's work, which is cited in *Science With Reason*, pp. 36–37.

- "Why do you think...?" (Asking about the learner's reasons)
- "How did you come to that conclusion?" (Asking about the validity of the learner's statements)
- "Couldn't it be right that...?" (Asking for supportive evidence)
- "How do you know that...?" (Asking for supportive evidence)
- "How might we find out whether...?" (Asking for alternative possibilities)

- **Use open-ended questioning techniques**. It may come as a surprise that such techniques are very difficult to develop. It takes significant planning and effort to master them. Many teachers have found these strategies helpful:

  - ✓ When conducting a discussion, **prepare several key questions** to get things rolling. This technique also gives teachers a checklist to help them make sure that key points are covered.
  - ✓ **Avoid** questions that can be answered with "**yes**" or "**no**."
  - ✓ Ask questions that **require students to think critically** to explain their observations and draw conclusions (e.g., avoid questions that let students provide only a number answer to an equation).
  - ✓ Encourage students to **use math language** by using it in your questions and asking follow-up questions that elicit the correct math terms when students neglect to use them in responses. (See Appendix C for math terminology.)
  - ✓ **Answer a student's question with a question**. Used in the appropriate circumstances, this technique encourages students to think more critically and to solve problems on their own (or with other classmates). *Hint: A curious, supportive demeanor is critical to the success of this technique. Many teachers have learned to start with a validating comment like, "That's a really good question, Jan," followed by a question.*
  - ✓ After asking individuals or groups of students a question, **allow 5–10 seconds of *wait time*** before talking, providing a hint, or calling on someone. This allows all students to respond—even those students who may be unsure of themselves and those who prefer to think answers through before talking about them.

✓ **Ask questions without bias.** Refer to the upcoming section concerning instructional equity for ideas such as:

- ? Pose the **same number of questions** to both boys and girls.
- ? Ask **questions with the same level of challenge** of both boys and girls.
- ? Follow both boys' **and** girls' answers to one question with a **second, more difficult question**.

◦ Encourage students to apply what they learn by encouraging them to **reflect** upon and explain their problem-solving process. Ask for such reflection with comments like:

- ? "How did you get your answer?"
- ? "Explain what you were thinking."
- ? "How do you know that your answer is correct?"
- ? "Can you make up a problem that is similar to this one?"
- ? "Draw a picture of what happened."
- ? "Can you show me or describe a model like it?"

Assigning journal entries is a great way to give students time to reflect upon their problem-solving processes.

Many of the items listed are modeled in *Fun with Math*. For example, hands-on learning and real-world applications are integrated into most lessons. Student-centered learning and most of the questioning techniques are found in class discussions within engagement and exploration activities, as well as in the evaluation techniques at the end of each lesson. To help deepen your understanding of inquiry-based instruction, look for the ways it has been integrated into this book's learning activities and apply any strategies you find helpful to other learning activities.

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Teaching Process Skills

Learning subject-area content is important. To become proficient in the competencies identified by math educators, students must understand basic math operations and principles. However, the ability to **apply** content is equally important. At work and at home, people need to apply their knowledge of the content in solving problems. **Process skills** lay the foundation for people to successfully apply content knowledge to solving problems in real-world situations.

Consequently, teachers of all levels need to consider process skills as they plan math instruction. When learning activities help students practice process skills, learning is more active and students learn more than if they were lectured to or if they observed demonstrations.

The Self-Check Worksheet on the following page defines the process skills generally used in math and science education. You may wish to use it to identify the process skills you currently teach and the ones you might want to add. Then, identify ways to help your students develop and/or practice these skills. Some ideas are provided on pp. 14–17.

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Process Skills Descriptions and Self-Check Worksheet

Instructions: In the first column, check the process skills that you currently use in instruction. In the second column, check those skills you'd like to add or expand upon. Use the resulting information as you choose learning activities from *Fun with Math.*

I Provide	I Want to Do More	
☐	☐	**Building models:** Constructing a representation of a system that is based on observations and inferences.
☐	☐	**Categorizing or classifying:** Arranging objects or systems into categories based on shared characteristics. Can also refer to labeling objects or systems based on unique characteristics or some other specified criteria.
☐	☐	**Communicating:** Conveying information (e.g., insights, explanations, results of observations or inferences, measurements) to others. Communication methods might include verbal, pictorial, graphic, or symbolic presentations.
☐	☐	**Comparing:** Relating one thing to another in order to identify similarities and differences.
☐	☐	**Controlling variables:** Holding all variables constant that impact an experiment or situation, except one variable whose influence is being investigated in order to evaluate changes in the others.
☐	☐	**Experimenting:** Testing a hypothesis through information gathering.
☐	☐	**Hypothesizing:** Forming precise questions to be tested scientifically. Formal hypotheses are stated so that each explanation may be tested and, based upon the results of those tests, accepted or denied.
☐	☐	**Inferring:** Suggesting explanations, reasons, or causes for observed events.
☐	☐	**Interpreting data:** Studying data, then summarizing its implications in the context of a scientific investigation. Familiar language should be used to describe the significance or meaning of data and observations.
☐	☐	**Measuring:** Using instruments to define objects or systems quantitatively, either as compared with others or as compared with a standard. Measuring includes the monitoring of changes in size, shape, position, and other properties.

I Provide	I Want to Do More	
☐	☐	**Observing:** Using the senses and extensions of the senses to closely examine or monitor a system, noting and recording aspects that are not usually apparent under casual scrutiny.
☐	☐	**Ordering:** Using observed characteristics to organize objects or systems in a sequence.
☐	☐	**Predicting:** Forecasting a future observation or the next occurrence in a system or series of events based on prior observations and inferences.
☐	☐	**Reasoning:** Making judgements based on observations, knowledge, and experiences. Usually involves making inferences and drawing conclusions.
☐	☐	**Recognizing relationships:** Interpreting interactions between different components of a system.
☐	☐	**Recording:** Creating a written record of observations made during experimentation.

Notes

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Strategies for helping students develop and practice process skills

The best way for students to learn process skills is for them to practice using those skills. Most of the time, process skills can be taught in conjunction with instruction about mathematical (and scientific) principles. Each learning activity in this book allows students to practice a wide variety of process skills along with content-area skills. (Refer to the first page of each activity to determine which process skills are targeted. In addition, a chart that identifies the process skills that are employed in each learning activity is located in Appendix E, pp. 428–440.)

However, there will probably be times when you want to give students opportunities to learn process skills **in isolation**. Let's use the process skill of **classification** as an example. Mathematicians and scientists put the objects they study into categories so they can more easily identify *similarities and differences*. As is the case with other process skills, classification skills are required in many job fields. They are increasingly important to daily living as we strive to create order and efficiency out of the multiple activities and demands that fill our days. Therefore, students can benefit from activities that help them learn classification skills in isolation by sorting objects with the same or different characteristics. (Even very young students can discern differences in characteristics such as shape, color, form, texture, and size.) Two activities that teachers can use to help students develop classification skills follow.

Classification Activity 1: Button Sort

Exploration activity

1. Give a pile of different buttons to each group of 2–4 students.

2. Challenge students to **identify categories** and **sort** the buttons accordingly. To help them focus on the observable characteristics of the buttons that can affect classification, you can ask questions like:

   ? How many buttons are there all together?

   ? How many shaded buttons are there?

   ? How many white buttons are there?

   ? How many of the buttons have 4 holes?

   ? How many white buttons have 4 holes?

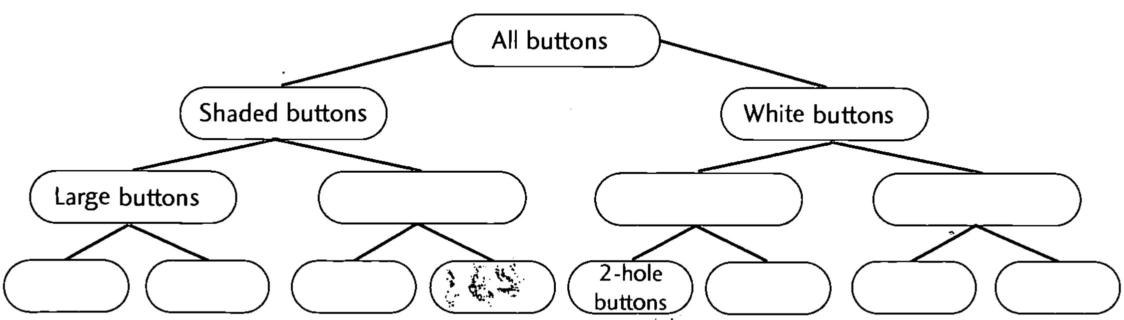
   ? How many shaded buttons have 2 holes?

   ? How many shaded buttons have 4 holes?

   ? How many buttons have 2 holes?

   ? How many large buttons have 2 holes?

3. Have students develop a **chart** to record their findings, similar to the illustration. Note: Depending upon the skills and experience of the students, have them work individually, in teams, or as a class to complete the chart. Skill differences will also determine whether or not the chart given to students contains any given information.



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Classification Activity 2: Observable Characteristics

Exploration activity

1. Give groups of 3–5 students items to classify (e.g., toys, fruits, vegetables, flowers, leaves, shells, buttons, sponges, counting manipulatives, stones, kitchen utensils, tools, cutouts of geometric shapes, natural vs. manufactured items, items of clothing).

2. Instruct individual students to examine each object, looking for its **observable characteristics** such as color, shape, texture, and size.

3. Have each student in the group **describe the characteristics** of 1 object to the other group members.

4. Instruct each group to sort all items into 2 piles by the similarities and differences of their observable characteristics. Examples: Sort all large objects into a pile and all small objects into another; all red objects into a pile and all non-red objects into another; all objects with wheels into a pile and all objects without wheels into another.

5. Have the students discuss the **criteria** that they used to sort the items.

6. Have students construct a **dichotomous key** (or chart) similar to the illustration in Classification Activity 1. They should classify objects until all items have been separated singularly. Note: When first introducing this process skill to students, the key can be developed as a class, with coaching from the teacher. Once students gain experience with this type of activity, they can construct charts independently and in team exercises.

Extension activities

- ☼ After groups complete their charts, introduce new, related objects (limit of 3). Instruct groups to *self-evaluate the results* by adding the new items to their charts.

- ☼ Have a representative from a group read a description (i.e., the characteristics) of an object on their chart to another group. Instruct classmates in the other group to identify the object.

* Place sets of different kinds of objects at different stations. Have groups of students rotate from station to station, developing a chart for each set. Then, have the groups compare and contrast the charts from all of the stations.

* Have each group member choose 5 items that share the same characteristics. Have others in the group identify which characteristics the items have in common.

Teaching Problem-Solving Skills

Problem-solving strategies are critical to any instruction aimed at improving real-life math skills. By using the learning activities in *Fun with Math*, you are giving students opportunities to develop their math problem-solving skills.

The ideas listed below should help you to provide students with additional opportunities for developing their problem-solving skills:

- Many **commercial games** require players to use problem-solving skills, including deductive reasoning, categorizing, visual discrimination, and/or calculating. Some games are listed below to help you get started. Several games on the list may not be available from local retailers. However, suppliers of instructional math materials, such as William Sheridan and Associates, which is listed in the Supplier section of Appendix B, pp. 408–417, should carry them.

  Abalone®
  Architek Game®
  Backgammon
  Battleship®
  Block by Block®
  Checkers
  Chess
  Chinese Checkers
  ConFIGURE®
  Connect Four®
  Connections®
  Continuo®
  Duo®
  Izzi® and Izzi 2®
  Logix®
  Mastermind® for Kids
  Mancala
  Othello®
  Quarto®
  Quick Chess
  Rummikub®
  My First Rummikub®
  Sequence®
  Shape by Shape®
  Take 6®
  Tangoes®
  Tangrams®
  Traverse®
  TriOminoes®

- Start several classes a week with a **brain teaser**. Refer to books such as *Brain Teasers!* (grades 1–6), *Favorite Problems*, and *Super Problems* or game card packs such as TOPS Communication Cards and Visual Brain Storms 1 and 2®[4] for ready-to-use activities. (Information about these and other books is provided in Appendix B.)

[4] Visual Brain Storms: The Smart Thinking Game® and Visual Brain Storms 2® are distributed by Binary Arts Corporation.

- Start several classes a week with a **math energizer** like the one below:[5]

  1. Pick a number from 1 to 9.
  2. Subtract 5.
  3. Multiply by 3.
  4. Square the number (or multiply the number by the same number).
  5. Add the digits until you have a single digit (e.g., 64 = 6 + 4 = 10 = 1 + 0 = 1).
  6. If the number is less than 5, add 5. If it is more than 5, subtract 4.
  7. Multiply by 2.
  8. Subtract 6.
  9. Map the digit into a letter in the alphabet (i.e., 1 = A, 2 = B, and so forth).
  10. Pick a name of a country that begins with that letter.
  11. Take the second letter in the country name and think of a mammal that begins with that letter.
  12. Think of the color of that mammal.
  13. Now share the color, name of the mammal, and country it is from with your neighbor.
  14. What did you discover? Can you explain what happened?

  (Solution: Each learner will have the same answer—an elephant from Denmark)

- **Have students solve real-life, hands-on problems.** Project-based learning activities are an extremely effective strategy for teaching real-life problem solving. Note that these activities may require students to work with one another—in the same way that they would in the workplace. And projects involve the integration of several subject areas (i.e., math, science, social studies, communication), which resembles real-world problem solving. In addition, students are required to figure out what information, materials, and tools they need. Your role as a teacher is to **facilitate learning**—*not* to tell students *what* to do and *how* to do it. (Refer to pp. 7–10 for suggestions about being a resource for students, leading discussions, and asking open-ended questions.) Examples of projects include the following:

  ✓ Have students calculate the area of classroom walls or floors to determine the amount of carpeting or wallpaper that would be needed to cover the surface. Then, have them determine the quantity of materials that should be purchased to complete the job. The activity can be made more complex by having them determine time and cost factors, and calculating the cost per unit.

[5] This problem was taken from the Internet (original source unknown).

✓ Have students design and build something from wood and nails (e.g., a bat house, bird house). In addition to calculating dimensions and measuring angles, they should determine the amount of materials needed and calculate construction costs.

✓ Have students operate a store. They should order materials, keep track of inventory, and calculate costs.

✓ Have students prepare and distribute a meal for the class. (Chili or soup would be a good choice.) They should determine quantities of ingredients needed to prepare each dish, calculate the amount of tableware needed, and figure out the per-person cost of the meal. Recipes can also be increased and/or decreased to feed a specified number of people.

✓ Challenge students to design and build a model of a putt-putt hole.

✓ Develop a "camping" experience that includes:

  - Pitching tents, then measuring angles, perimeters, areas, and volumes.
  - Stacking firewood, then identifying patterns.
  - Identifying patterns found in nature.
  - Answering real-life questions, such as:
    - ? How many stones are needed to surround the fire circle?
    - ? How many air mattresses (or sleeping bags) will fit in each tent?

Project-based learning activities can also help teachers by providing an effective way to assess a student's ability to *apply* mathematical principles to real-life situations. For example, a teacher may challenge a group of students to solve a real-life problem. The teacher might then make and note observations during the problem-solving process and/or may use a rubric to evaluate students' skills in mathematical problem solving. Assessment is discussed in more detail on pp. 28–36, including a sample rubric.

Employing Cooperative/Collaborative Learning

Cooperative/collaborative learning is an instructional method that gives students opportunities to **work interdependently to accomplish an academic goal, while still being held individually accountable for learning**. There are many benefits to this style of instruction, including the following:

- Students take responsibility for their own learning.
- Students learn to combine their critical thinking skills to solve problems, resulting in less dependence on the teacher and increased student-centered learning.
- Students develop leadership skills.
- Students learn and develop social skills.
- Students learn to share responsibility.
- Students experience enhanced self-esteem.
- Students build trust in others.

Cooperative/collaborative learning groups can be made up of pairs or small numbers of students heterogeneously grouped. Here are suggestions for structuring cooperative learning activities:

1. Assign each group member a specific **role** (e.g., reporter, recorder, leader, materials monitor) and specify the responsibilities of each role. Students may have more than one role, depending on the number of students in each group and the number of roles required.

Hint: Some teachers have students develop job descriptions for each role.

2. Define the **skills** students should practice (e.g., talking quietly, listening, staying on task).
3. Present a **task** or **activity**, and give directions and guidelines to each group.
4. While students are working collaboratively, **monitor** and **assess** group and individual behaviors.
5. After the tasks are completed, instruct group members to **evaluate** their group's progress, and their performance as team members.

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Some effective cooperative/collaborative learning strategies

The following strategies can help you implement collaborative learning:

- **Think-Pair-Share:** A problem is posed. The students think alone about the question for a specified amount of time, then form pairs to discuss the question. During share time, teammates are called upon to share the answer with the class.

Hint: This method is helpful for sharing how a math problem was solved or which homework problem was most difficult and why.

- **Three-Step Interview:** This strategy works best with groups of 4 students, but can be used with larger or smaller groups. Students form pairs within their groups. One member in each pair is the interviewer and the other the interviewee. After interviewing, the pairs reverse roles and complete the interview again. Then, the pairs take turns sharing what they learned in their interviews with the other members of their team.

- **Rallytable:** Students work in pairs within groups of 4. They pass a sheet of paper back and forth, recording answers to a problem that has many answers. When time is called, the pairs compare their answers within their groups.

- **Send-A-Problem:** Each student on a team makes up a review problem and writes it down on a flash card. The author of each question asks it of his team members. If there is consensus, the author writes the answer on the back of the card. If not, the question is revised so it produces consensus. The question side of the card is marked with a "Q," and the answer side of the card is marked with an "A." Each team passes its stack of review problems to another team. One student reads the first problem to the team, who attempts to answer it. If team members have consensus, they turn the card over to see if they agreed with the sending team. If not, they add their answer to the answer side of the card. Each question is handled in this way, then the stack of cards can be passed on to other teams. When the cards are finally returned to the authoring team, the team members can discuss and clarify the questions and answers based on the alternative answers recorded by other teams.

- **Co-op Jigsaw 1—Experts Report:** Each student becomes an "expert" for an assigned topic. Each expert meets with experts on the same topic from other teams to learn basic principles and information. Each group of *experts* makes a presentation to the class. When students return to their teams, the varied experts combine their information to create a group product. Example:

  ✓ Topic: measurement
  ✓ Team topics: a baseball, a golf ball, a brick, an eraser
  ✓ Expert topics: height, weight, volume, shape

Each expert team presents information about its measurement topic to the class. Then, experts return to their groups to combine their measurement knowledge in completing an assignment regarding their object (e.g., baseball, golf ball, brick, eraser).

- **Co-op Jigsaw 2—Team Reports:** This strategy is similar to Co-op Jigsaw 1, except experts do not report to the class. Instead, *teams* report to the class, each on a different topic. The assigned experts from each group meet to learn basic principles and information, then return to their teams and apply those principles and information to a specific problem. When team projects are complete, each team presents its project to the class. Example:

  ✓ Topic: measurement
  ✓ Team topics: a baseball, a golf ball, a brick, an eraser
  ✓ Expert topics: height, weight, volume, shape

  After students learn their respective measurement skills in their expert groups, each expert teaches his/her teammates the measurement skills learned. Then, teammates apply those skills to their group's object (e.g., baseball, golf ball, brick, eraser) and complete an assigned project together. Finally, each team reports to the class.

Providing Equity in Instruction— What Teachers Can Do

An often-overlooked aspect of instruction is **equity**—providing instructional opportunities that will help **all** students succeed. It is especially important for teachers to consider equity when planning lessons related to math, science, and technology skills because these subject areas are especially vulnerable to unconscious biases that impact the quality of education provided to some student groups (e.g., females, socially-disadvantaged students, differently abled students). In this section, we'll use male-female equity for illustration purposes, but the principles and techniques discussed can be applied successfully in any situation to ensure equity in instruction for **all** students.

When it comes to math, science, and technology, many girls do not have the same types of experiences, both in and out of school, as boys. As a result, boys and girls have traditionally received differing qualities of education—even when they study in the same classroom. It's possible that you are skeptical about this—especially if you are male. Please don't stop reading; the information provided in this section will help you quickly and easily check it out for yourself.

Discovering unconscious biases

Researchers[6] have found the following inequities in the way that boys and girls are treated in school:

- At all grade levels, girls **receive less teacher attention and less useful teacher feedback** than boys do.

  ✓ Teachers **call on** girls less often than they call on boys.

  ✓ Teachers **ask probing and higher-order thinking questions** of girls less often than they do of boys.

  ✓ Teachers often instruct boys on **how** to perform tasks but tend to **do** tasks for girls.

  ✓ Teachers tend to **give less feedback** (e.g., praise, criticism) to girls than they do boys.

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[6] Myra and David Sadker have done the most comprehensive research on the topic of gender equity. Their findings are published in *Failing at Fairness: How America's Schools Cheat Girls* (1994).

- In class, **girls talk significantly less** than boys do.

  ✓ In elementary and secondary school, girls are eight times less likely to **call out comments**. When they do, they are often reminded to raise their hands, while similar behavior by boys is accepted.

  ✓ Girls are less likely to **raise their hands** because they are aware that boys get called on more, they may take longer than boys to think about their responses before raising their hands, and they may not have confidence in their ability to answer correctly.

- When working in coed pairs, **boys tend to dominate** in math, science, and technology-related activities, which results in girls having fewer opportunities to experience hands-on learning in these areas.

- Girls rarely see **women's contributions to math, science, and technology** mentioned in the curriculum. Most textbooks continue to report only male contributions and may still contain pictures of males actively doing things while females passively watch and support them.

- Girls experience **pressure from their friends to *not* do well** in science, math, and technology-related classes; they may be teased for being "nerdy" or unfeminine if they try to do well.

Outside the classroom, most girls have not received opportunities to work with tools or mechanical systems. Girls don't have as many toys that encourage them to build, explore, or tinker. For example, girls do not generally build with LEGOs® or fix their own bicycles. And as in school, adults tend to ***instruct* boys on how to perform tasks** but tend to ***do* tasks for girls**. As a result, many girls have not developed the foundational skills needed to succeed in many real-world problem-solving situations.

This subconscious inequity has far-reaching consequences. Because girls have had fewer opportunities to develop mechanical skills than boys, they are often less prepared for and less likely to take courses in math, science, and technology. In addition, it is clear from the research that those girls who do take upper-level math and science courses are treated very differently than their male classmates, causing many to stop taking them. Taking fewer of these courses has resulted in fewer high-paying occupational options for female workers.

Strategies for overcoming biases

What can teachers do to help **all** students receive equitable, *high-quality* educational experiences in math and science? The following list provides some ideas.

- The most important thing you can do, as a male or female teacher, is to **be aware of your subconscious biases** and to accept that *virtually everyone* has been socialized to have biases. (If you don't believe that you treat some students differently—boys and girls for example—ask a colleague to observe you in the classroom or make a videotape of your teaching to determine whether or not you provide girls with less attention and different expectations than you do boys.)

- Make a conscious effort to **provide girls and boys with equal amounts of attention and encouragement**. Transfer this conscious effort to any other student groups that may be receiving less attention.

- **Create an atmosphere that fosters girls' participation**. Because girls are generally more shy than boys about speaking in public, you can make it more likely that girls will express themselves by making small changes in your own behavior. For example, when asking questions in coed settings, make a conscious effort to wait 5–10 seconds before calling on anyone. You'll be surprised at the number of timid hands that go up in those few seconds of *wait time*. Or, if only 1/3 of the students have their hands up after you ask a question, you might want to say, "Think about it and talk with the person sitting next to you. I'll ask again in a few minutes." This technique is an especially easy and effective way to level the playing field for **all** students.

- **Pay attention to group dynamics**—who speaks, how often, for how long, in what order, and who interrupts whom. Actively facilitate discussions in ways that give **all** students a chance to participate. For example, since girls tend to speak less often and for shorter periods of time than boys do, be sure to acknowledge their contributions.

- **Don't "teach down" to any students**. Expect equally high performances from boys and girls and from people of all races, national origins, and abilities. For example, avoid the assumption that a girl cannot use tools or analyze complicated systems.

- **Involve girls in construction and manipulation of equipment and the use of tools**. They may not have had these types of experiences in the past. For example, many girls have not had opportunities to use pliers, pipe wrenches, handsaws, electric drills, ratchet sets, ohmmeters, micrometers, gauges, or bench vises. Teachers should provide opportunities for them to learn these skills. Socially

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disadvantaged and differently abled students often have not been given these types of experiences either, so be sure to include them in such activities.

- Provide girls and other student groups with **additional instruction or practice**, when needed.

- **Use a collaborative, cooperative approach to activities** rather than a competitive one.

- **Use interactive methods**. Have students do some of their work in small groups. Girls tend to be more effective when they can share their ideas with others. Note: In the beginning, provide additional facilitation for groups that contain girls and other student groups to encourage their participation. Most students tend to change their interactions with others very quickly once they see different methods modeled by an adult, because their unconscious biases are still pliable and they instinctively desire to include others. Therefore, additional facilitation will become unnecessary within a short time.

- To prevent boys from dominating activities in which they have more skill and experience, **pair girls with girls and boys with boys** some of the time. Making careful pairing choices works well with other student groups too.

- **Foster students' independence**. For example, hold girls accountable when they engage in "learned helpless" behavior. If they say, "I can't do it" before exerting effort, find ways to re-engage them in the activity. Show faith in **all** students' abilities to do things for themselves.

- **Use more than one method of assessing student achievement**. For instance, do assessments using multiple-choice tests and model building (at which boys tend to perform better) as well as essay tests, projects, and reports (at which girls tend to perform better).

Don't let this long list overwhelm you! Just pick one strategy, try it for a week or two, then try another one. Your efforts will surely pay off for **all** of your students.

Using Assessment as a Teaching Tool

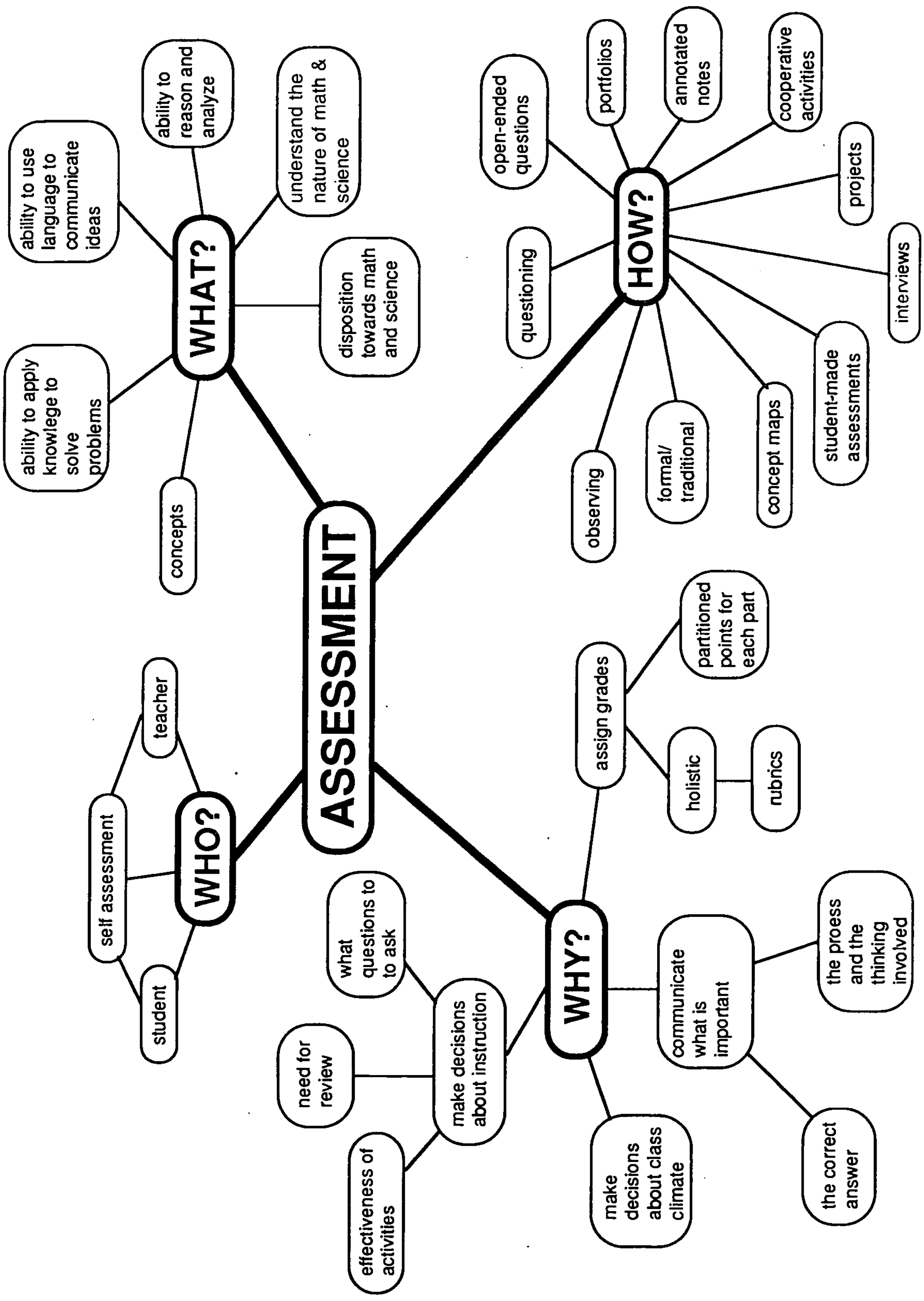
Assessment is a key component of successful math instruction. Measuring students' understanding of math content and processes *before, during,* and *after* instruction is critical to ensuring that future instruction meets students' needs, and that your learning goals are being met.

Assessments that give teachers insights into students' thinking and the development of students' conceptual growth are the most valuable. In addition to helping teachers assess student learning, they help teachers decide in what experiences to engage students next.

Whenever planning assessment, teachers should ask these questions:

- **Who** is being assessed?
- **Why** am I assessing?
- **What** am I assessing?
- **How** am I going to assess?

The concept map on the following page identifies many aspects involved in planning for effective assessment.


ASSESSMENT
WHO?
self assessment
teacher
student
WHAT?
ability to apply knowlege to solve problems
concepts
ability to use language to communicate ideas
ability to reason and analyze
understand the nature of math & science
disposition towards math and science
HOW?
open-ended questions
portfolios
annotated notes
cooperative activities
projects
interviews
student-made assessments
concept maps
formal/ traditional
observing
questioning
WHY?
assign grades
partitioned points for each part
holistic
rubrics
make decisions about instruction
what questions to ask
need for review
effectiveness of activities
communicate what is important
the proess and the thinking involved
the correct answer
make decisions about class climate

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Assessment strategies

Assessment strategies can vary—depending on the age and skill levels of the students and instructional style of the teacher. Many assessment methods are effective, including the following:

- Students can better assimilate what they learn by **journaling**. Through journaling, they *reflect* upon the process by which they explored math principles, made generalizations, and came to conclusions about what they experienced. In addition, journal entries help students *apply* what they've learned to their daily lives at home, at school, and in the community. Teachers who encourage their students to write about their mathematical procedures, strategies, and techniques send the message that the student's *thinking skills* are just as valuable as the correct answer.

- Student and teacher **portfolios** allow both students and teachers to select and save materials that show students' progress over time. Portfolios can contain required pieces, individually selected pieces, and unit projects. Two important parts of this assessment are a chart of growth (or progress chart) and an explanation of why each item is included.

- Student **presentations**, during which individuals or teams of students present their findings and conclusions to others, help students develop a variety of real-world skills (e.g., communication, critical thinking, collaboration). When teachers facilitate discussions after such presentations, students learn to listen to feedback and how to apply it to their conclusions, revising them if needed. (See the strategies for such discussions in the Teaching Through Inquiry section, pp. 5–10.)

- **Student interviews or conferences** with a teacher allow them to describe their understanding of what they learned and to make applications to everyday life.

- **Quizzes** can check students' knowledge of mathematical concepts. Note: Paper-and-pencil tests rarely assess students' abilities to employ process skills.

- **Rubrics** are especially helpful in evaluating students' work on design projects, which allow them to apply mathematical principles to real-world situations. Rubrics can help organize thinking about assessment and guide your observations and recording. See sample rubrics on pp. 35–36. Note: Rubrics are best used in conjunction with other types of assessment.

- **Teacher observation** of students as they solve problems gives teachers a view into students' thinking processes. For example, teachers can assess students' abilities to use process skills and apply content-area knowledge by observing students as they complete the learning activities in *Fun with Math*. The sample Observation Sheet on pp. 32–34 can be used for this purpose.

  The Observation Sheet is designed for teachers to record a student's:

  ✓ Content-area knowledge.

  ✓ Ability to employ problem-solving strategies.

  ✓ Ability to employ process skills.

  ✓ Overall attitudes and skills.

  This assessment tool can be used effectively in the classroom to:

  ✓ Guide the teacher's observation of specific skills as they relate to a content area.

  ✓ Document each student's progress.

  ✓ Measure the amount of progress by comparing each student's work to his or her previous work.

  This formal observation strategy results in a complete and detailed written assessment of each student's individual progress that can be used for grading and for planning future instruction. These sheets can also be useful in organizing homogeneous or heterogeneous groups for small group instruction.

Observation Sheet

Instructions:

1. Make one copy of this sheet for each student being assessed.
2. Label each sheet with the student's name and the unit or content topic name.
3. Choose the outcomes you wish to assess. Mark targeted process skills, write in problem-solving skills, and identify general observation questions.
4. Observe students during math problem-solving activities—before, during, and after a unit instruction—and record observations. *(Hint: Using a different color of ink to record the data for each observation date will make comparisons easier.)*

Reminder: Be sure to record the observation date.

Student Name ______________________ Unit/Content _______

Before __________ (date) During _________ (date) After ________ (date)

Process Skills	Completes Independently	Completes with Assistance	Cannot Complete
Building models: Constructing a representation of a system that is based on observations and inferences.			
Categorizing or classifying: Arranging objects or systems into categories based on shared characteristics. Can also refer to labeling objects or systems based on unique characteristics or some other specified criteria.			
Communicating: Conveying information (e.g., insights, explanations, results of observations or inferences, measurements) to others. Communication methods might include verbal, pictorial, graphic, or symbolic presentations.			
Comparing: Relating one thing to another in order to identify similarities and differences.			
Controlling variables: Holding all variables constant that impact an experiment or situation, except one variable whose influence is being investigated in order to evaluate changes in the others.			
Experimenting: Testing a hypothesis through information gathering.			
Hypothesizing: Forming precise questions to be tested scientifically. Formal hypotheses are stated so that each explanation may be tested and, based upon the results of those tests, accepted or denied.			
Inferring: Suggesting explanations, reasons, or causes for observed events.			

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Observation Sheet, Page 2

Process Skills	**Completes Independently**	**Completes with Assistance**	**Cannot Complete**
Interpreting data: Studying data, then summarizing its implications in the context of a scientific investigation. Familiar language should be used to describe the significance or meaning of data and observations.			
Measuring: Using instruments to define objects or systems quantitatively, either as compared with others or as compared with a standard. Measuring includes the monitoring of changes in size, shape, position, and other properties.			
Observing: Using the senses and extensions of the senses to closely examine or monitor a system, noting and recording aspects that are not usually apparent under casual scrutiny.			
Ordering: Using observed characteristics to organize objects or systems in a sequence.			
Predicting: Forecasting a future observation or the next occurrence in a system or series of events based on prior observations and inferences.			
Reasoning: Making judgements based on observations, knowledge, and experiences. Usually involves making inferences and drawing conclusions.			
Recognizing relationships: Interpreting interactions between different components of a system.			
Recording: Creating a written record of observations made during experimentation.			
Problem-Solving Skill:			
Problem-Solving Skill:			
Problem-Solving Skill:			
Problem-Solving Skill:			
Math Skill:			
Math Skill:			
Math Skill:			
Math Skill:			
Math Skill:			

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Key Questions for General Observations

How did the student participate with teammates?

Did the student assume a role in the group? What role?

What did the student do?

What did the student have problems with?

What did the student do when difficulties were encountered?

If there is a product, does it meet the parameters outlined in the problem?

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Fun with Math Sample Rubric #1
(For Projects)

	1–Unacceptable	2–Marginal	3–Acceptable	4–Exemplary
Depth of Thought	Major gaps are evident. Little or no reasoning is demonstrated.	There are major gaps in reasoning. Reasoning is somewhat apparent, but is flawed.	Reasoning is apparent, but a few minor gaps or flaws exist.	Reasoning is clear, concise, and effectively demonstrated.
Presentation	Written or oral presentation is characterized by haphazard, sloppy, or missing information. Written report is not typed.	The presentation lacks major points of emphasis and/or information is not provided in a professional manner.	The presentation is pleasant, pleasing, and informative and is clearly designed around informing the intended audience.	The presentation mimics professional quality. The message is clearly articulated to the intended audience.
Feasibility	The project solution is clearly not possible within the parameters set forth by the problem.	The feasibility was in question until an explanation was requested and given. The solution may not be possible within the parameters of the problem.	While the solution is valid, it may not be easily replicated.	It is clear that the method of solution is valid and can be easily replicated.
Attention to Detail	The project is generally characterized by superfluous or surface knowledge.	Only a few questions are answered in detail. The work generally does not attend to the underlying detail required by the problem.	Most of the questions posed by the problem are directly answered in detail.	Questions are anticipated and addressed. All measures, scales, and other required annotations are documented.
Creativity	The approach to the project is a direct replication of a previous design. No new ideas are demonstrated.	The approach is obviously related to a previous design, but some novelty is shown.	While the design presented may be similar in approach to others, unique characteristics exist that make this design stand out.	The approach to the design is fresh, novel, and unique.

Fun with Math Sample Rubric #2*

	1–Unacceptable	*2–Poor*	*3–Marginal*	*4–Satisfactory*	*5–Exemplary*
Were effective problem-solving strategies used?	Student did not use strategies.	Student tried one or more strategies, which were not effective.	Student used some effective strategies.	Student employed very effective strategies.	Student employed creative, efficient strategies.
Is work accurate?	Math was completely inaccurate.	Math was mostly inaccurate.	Math was mostly accurate.	Math was almost completely accurate.	Math was completely accurate.
Is work shown?	No work was shown.	Very little work was shown.	Some work was shown.	Most work was shown.	All work was shown.
Was the student an effective teammate?	Student did not participate or cooperate with others.	Student did not cooperate with others.	Student sometimes cooperated with others.	Student was generally cooperative.	Student was very cooperative; listened and worked together with others.
Is the student's work easy to understand?	No answers were given.	Answers were not readable or clear.	Answers were readable but not completely clear.	Answers were neat, clear, and understandable.	Answers were very neat, very clear, and well-written.
Could the student work independently?	Student refused to work alone or with the teacher.	Student needed teacher's help on all parts.	Student needed teacher's help on some parts.	Minimal teacher help was required.	Student worked independently when appropriate.

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* Adapted from *Special Delivery: Putting Math to Work* (1998) by Bob Krech. NY: Cuisenaire of America.

Effective Instructional Strategies for Teaching Real-Life Math Problem Solving

About Solving Mathematical Problems

We frequently encounter math-related problems in our daily activities—from the kitchen to sports to the bank to the grocery store. Math problem-solving situations are all around us. However, in many cases, the real-life problems we encounter differ from those found in textbooks, as illustrated in the comparisons below.

Textbook Math Problems	Real-Life Math Problems
Problems usually require only one step.	Problems usually require more than one step.
Problems usually require only one operation (i.e., +, −, x, ÷).	Problems often require more than one operation, which the problem-solver must identify.
There is usually only one right answer, and one suggested way to solve a problem.	Multiple solution possibilities exist. Choosing the best solution often requires a judgment call.
All needed information is provided, and is presented in the order in which it is needed.	Some information is missing or extra information is provided but not needed to reach a solution.
Answers are usually even (e.g., whole numbers).	Answers are often uneven (e.g., fractions, decimals).

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Steps for effective math problem solving

When participating in inquiry-based learning situations, students need to employ the problem-solving process. The steps in the **mathematical problem-solving process** are separate, yet interrelated. They are:

1. Understand the problem by answering, "What is the question?"
2. Find the facts needed to solve the problem.
3. Design and implement a solution strategy, taking into account the alternatives and given conditions.
4. Evaluate the results for reasonableness.

Each step is described in this section.

1. **Understand the problem by answering, "What is the question?"**
   To successfully develop real-life problem-solving skills, students must understand—in detail—the problems with which they are confronted. Useful techniques for helping students understand the conditions and facts related to a given problem include the following:

   - If students encounter **unfamiliar math terms**, have them learn their meanings. (Refer to the lists of math vocabulary and definitions in Appendix C, pp. 418–421, for suggestions.)

   - Teach students to **restate the problem in their own words.** *(Hint: Consistently modeling this technique is one of the most effective ways to help students learn to use it.)*

   - Have students **look for key words**, paying attention to context. Note: It may be helpful to instruct students to underline such words.

2. **Find the facts needed to solve the problem.** This involves:

   - **Identifying the key facts** by teaching students to list the given information, including limitations or criteria.

   - **Identifying the facts that aren't needed** by teaching students to cross out any unnecessary information.

   - **Determining if more facts are needed.**

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3. **Design and implement a solution strategy, taking into account the alternatives and given conditions.**

   a. Teach and/or model effective **solution designs** by doing some or all of the following with your class:

   - Suggest that students **think of a similar problem** that they've solved before and apply that experience to the current problem.
   - When completing a problem as a class, **draw a chart, graph, diagram, or model** that includes the information presented in the problem. Have students do the same; group activities and journal assignments are especially good opportunities to use this technique.
   - **Use manipulatives or dramatize the problem** during class discussions. Again, give students opportunities to practice the same techniques.

   b. Teach students to use some or all of the following **strategies** when they solve mathematical problems.

   - Work backwards.
   - Simplify the problem (e.g., reduce the level of complexity, use smaller numbers).
   - Break it into smaller problems.
   - Think of a similar problem.
   - Translate the problem directly into mathematical symbols.
   - Conduct an experiment, being sure to keep accurate records.
   - Design a physical model.
   - Make a sketch or diagram.
   - Act out the problem.
   - Use manipulatives.
   - Collect data and organize it into a table.
   - Construct a graph or make a tally.
   - Use grids or arrays.
   - Look for a pattern.

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- Close your eyes and visualize part or all of the problem
- Use deductive reasoning to eliminate solutions that are impossible or unrealistic.
- Develop a general rule.
- Use a trial-and-error method (sometimes called guesstimating or guess and check).

c. Be sure to emphasize and reinforce the importance of **reflecting upon and adjusting strategies throughout the problem-solving process**. Students must grasp the fact that as they learn more about the problem, they can add elements to the solution strategy. They should also be encouraged to select another solution path if their chosen strategy fails.

4. **Evaluate the Results for Reasonableness**
Students should examine their problem solutions for accuracy. You can help them practice this step by encouraging them to ask themselves questions like:

? Does the solution make sense?

? Did I use the correct units?

? Are there possible answers I didn't consider?"

? Is it practical, considering the context of the problem?

These questions can often be answered using strategies of estimation or approximation.

If new information is found during this stage, students should use that information to **refine their results**, redoing previous steps if needed. In addition, students should be encouraged to reflect upon problems that have been or could be solved with similar strategies during this step.

The best way to teach students about the problem-solving process is to help them **experience** it and then **reflect** upon it. *Fun with Math* was developed to give you a variety of math problem-solving activities that encourage hands-on learning experiences and student reflection. As you use this resource, look for ways to integrate its instructional strategies for real-world problem solving into your favorite math lessons. Also, Appendix B lists other resources that can help students develop their mathematical problem-solving skills.

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Fun with Math:
Real-Life Problem Solving for Grades 4–8

Strand I: Patterns, Relations, and Functions

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Notes

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Did Someone Say, "Patterns"?

About this learning activity

Students will use color patterns to learn about number patterns, then practice finding the missing number(s) in various number patterns.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I P	III ○	V ○	VII ○
II ○	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

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Materials

- A patchwork quilt (Use an actual quilt, a photograph, or a magazine.)
- Rainbow centimeter cubes or paper squares in various colors (a set of 20 for each student)
- Paper and pencils
- Overhead projector (optional)

Engagement activity

Ask students if they are familiar with patchwork quilts. Display a patchwork quilt and help students identify the patterns in the design. Discuss the possible steps needed to design the quilt, pointing out how **repetition** (following the pattern over and over) helps in designing a quilt.

Exploration activity

1. Distribute cubes or squares to student pairs. Instruct each student to **create a pattern** using 10 cubes or squares.

2. Have each partner study the other's pattern and continue it with 10 more cubes or squares. Note: Let students move around the room to observe other students' patterns.

3. Help students explain how they used **repetition** in both creating and continuing their patterns.

4. Explain to students that patterns are used in math too. Use skip counting (e.g., by 2, 3, 5) as an example, asking if anyone can explain the concept of skip counting.

5. Write the numbers *1, 3, 4, 6, 7, 9* on the board or an overhead transparency. Facilitate a discussion of this number pattern with these and other questions:

   ? What 2 numbers come next? Note: Have students write them on their papers.

   ? Can anyone describe the pattern in this sequence?

6. Repeat step 5, using these numbers: 15, 13, 12, 10, 9, 7.

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7. Ask 2 student volunteers to display and explain 2 additional number patterns. Have the first student use addition and the second use subtraction.

8. Display simple number patterns using multiplication and division, asking students to describe each pattern and continue it on their papers.

9. Display the following sequence: 4, 9, ___, 19, 24. Have each student copy the sequence on a clean sheet of paper.

10. In their student pairs, have them discuss the pattern and determine the missing number.

11. Provide additional sequences with 1 or more missing numbers. Let students repeat steps 9 and 10 for practice.

Hint: You can have students write number sequences and have classmates solve them.

Evaluation

Review the papers students completed during the exploration activity.

Journal assignment
Have students write about how patterns are important in our daily lives.

Extension activity

- Prepare a worksheet of picture or color patterns for students to complete.

Connections to other subjects

Art. Have students design a quilt or wall hanging that includes a pattern.

Music. Instruct students to find examples of patterns in musical selections of their choice, then play them for the class via their own instruments or a tape recording.

Social Studies. Discuss with students that time lines are written in sequence, then demonstrate a few. Let students create their own sequential time lines, giving extra points for creativity.

Language Arts.
- Point out patterns in poetry to students. Have them compose poems that contain patterns.

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- Use books from the following list for book reports, class discussions, and similar language arts activities.

  - 🕮 *Frog Math Power—Level C* by Mary Jo Hand (Frog Publications, 1997) (Game no. 16)
  - 🕮 *Mad Math* by Sue Macy (Scholastic, Inc., 1987) (p. 35)

Home connections

Have students look for patterns in their homes (e.g., bedding, wallpaper, decorations). Instruct them to sketch the patterns they find. Use the sketches for a school display.

Resources for teachers

Patchwork Math 2 by Debra Baycura (Scholastic, Inc., 1990)
Big Book of Everything—Fifth Grade by Mel Fuller (Instructional Fair, 1995) (pp.103–105)
Everyday Math—Grades 4–6 by Marge Lindskog (Frank Schaffer Publications, Inc., 1996) (pp. 13, 67, 71, 80)
Hundreds of Board Activities by Marilyn Preddy (Carson-Dellosa Publications, 1995) (pp. 38–43)
Mathematics Problem Solving Activities by W.G. Quast and Robert Willcutt (Houghton Mifflin Company, 1978) (pp. 34–37)
Math Activities by Robyn Silbey (Frank Schaffer Publications, Inc., 1996) (p. 16)
The Intermediate Mailbox, The Education Center, Inc., Greensboro, NC, August/September 1994–95 (pp. 36–42)

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Patterns, Patterns

About this learning activity

Students will practice identifying patterns in a variety of situations, then apply what they discover to several types of number patterns, including the n^{th} number.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I P	III ○	V ○	VII ○
II ○	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

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Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- A variety of buttons (or colored bingo chips, beans, magic markers, or similar objects)

Engagement activities

1. Use the buttons to create a sample pattern on poster board. Have students identify the pattern. Example: 4 blue buttons, 2 red buttons, 1 black button, 4 blue buttons, 2 red buttons. What comes next?

2. Play a name game. Begin by telling the students, "We're going on a class trip. I will bring potato chips and hamburgers. My name is Pat Handelman." Instruct each student to tell everyone what he or she would like to bring for the picnic. As students respond, tell those students who follow the pattern that they may go to the picnic and tell those who do not follow the pattern that they must stay behind. Note: Generally, all students will recognize the pattern after a few rounds. *(Hint: Adapt this activity for younger students by using first names only.)*

3. Tell students that patterns can be found all around us. Have them look around the room and/or school building and identify patterns (e.g., window pattern, door pattern).

4. Organize some students into a pattern in front of the class. Ask students to identify the pattern. Example: 2 boys with brown hair, 1 girl with black hair, 1 girl with red hair, 2 boys with brown hair. Who would fit into this pattern next?

Exploration activity

1. Give students a variety of buttons. Have each student create a pattern, then work in pairs to identify each other's patterns.

2. Switch from buttons to numbers. Write a number pattern on the board and have students identify it. Examples:

   - 1, 2, 3, 4, 5, 6. What's the next number?
   - 101, 103, 105, 107. What's the next number?

3. Create more advanced number patterns, leaving out a number in the middle. Have students identify the missing number. Example: 7, 9, 11, ____, 15, 17. What's the missing number?

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4. Have students create charts in which they identify a pattern, then project to the n^{th} number. Example: 1, 3, 5, 7, 9. The n^{th} term would be $2n - 1$.

n	?
1	1
2	3
3	5
4	7
5	9
n	?

Explanation

Students should be able to recognize simple patterns, such as numbers increasing by 1, 2, or 3. Example: 1, 2, 3, 4, 5, 6. What's the next number? In addition, students should be able to identify the pattern and write simple equations for finding the n^{th} term.

Evaluation

Have students create patterns for their classmates to solve. Then, instruct students to explain the patterns they solve in their own words. Example: 102, 106, 104, 110. The pattern for this problem is +4, −2.

Journal assignment
Have students explain some of the strategies they can use to determine patterns. Then, tell students to write an example of a numerical pattern.

Extension activities

- Have students order events from a story (i.e., what happened first, second, third, and so forth).

- Give students more advanced number patterns and let them practice identifying the pattern and the n^{th} number. Example: 1, 5, 7, 9, 1, 5, 7, 9, 1, 5. Identify the 21^{st} number. Answer: Students should recognize that the pattern has 4 numbers, so they need to divide 21 by 4. They get 5 and 1 left over. Therefore, the 21^{st} number will be 1 (the first number in the series).

☼ Repeat the previous extension activity, but instruct students to find the 27th number. Answer: Divide 27 by 4 to get 6 and 3 left over. The 27th number will be 7 (the third number in the series).

☼ Assign inductive and deductive reasoning problems. [Problems are available in *Math Mindbenders, Deductive Reasoning in Mathematics* by Anita Harnadek (Midwest Publications, 1989).]

☼ Have students look for patterns in each of their other classes, and make a list to display in the Math room.

Home connections

Have students list patterns found in their homes. Make a class list to display in the classroom.

Resources for teachers

Mathercise by Michael Serra (Key Curriculum Press, 1992)
Strategies for Math Problem Solving by Mosenfelder & Kaplan (Educational Design Inc., 1996)
Super Solvers Outnumbered! by The Learning Company (computer software for students)

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Rubber Band Enlargements

About this learning activity

Students will lengthen the sides of several shapes, then measure the impact of the change on the area of each shape.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I P	III ○	V ○	VII ○
II ○	IV X	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

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Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Small rubber bands (1 pair per student; each pair must be the same size.)
- Rulers
- Scissors
- Pencils
- Tape
- Enlargement Worksheets 1 through 4

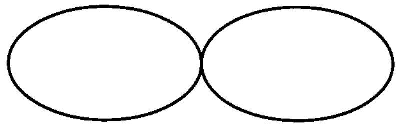
Engagement activity

This learning activity does not require an engagement activity.

Exploration activity

Hint: Try this activity yourself before working with the students so that you can coach them through any difficulties. You may wish to demonstrate the procedures on an overhead, using Worksheet 1, then allow students to practice with Worksheets 2 through 4.

1. Instruct the students to tie together the 2 rubber bands to make a shape similar to a figure 8 (see illustration).



2. Have students work in pairs. Explain that teams will be enlarging the shapes on the worksheets using pencils and their rubber bands.

3. Tell students that some tips (listed below) can make this task a little easier, then write them on the board. Explain and/or demonstrate each tip.

   - Tape the paper to the desk so it does not move.

   - Hold the pencil as perpendicular to the desk as possible.

   - Have the rubber band as close to the pencil tip as you can.

4. Instruct teams to complete Worksheet 1 by having 1 student hold 1 rubber band opposite the knot on the desk while the second student places a pencil in the second rubber band opposite the knot.

5. Tell students to stretch the rubber bands so the knot traces the square while the pencil tip draws a corresponding shape.

6. Have teams measure both the original square and the square they drew, then record the measurements on their worksheets. Note: The sides of the drawn squares will be twice the length of the sides in the original worksheet square.

7. Instruct teams to cut out the original square and place it inside the drawn square. Tell teams to cut more squares in the original size until they completely fill the drawn square. (Answer: It will take 4 squares.)

8. Have teams repeat this activity with the other 3 worksheets.

9. Facilitate a class discussion with these and other questions:

   - How many of the original shapes did it take to fill each of the drawn shapes? (Answer: 4)
   - Why did it take 4?
   - Will it always take 4?
   - What relationship exists between the **side lengths** of the originals and the drawn figures?
   - What relationship exists between the **areas** of the originals and the drawn figures?

Explanation

Students should recognize that they can create the new figure by cutting out 4 copies of the original object and fitting them together. Then, they should discover that the area of the original is multiplied (enlarged) by 4. Finally, they should switch from using drawings to calculate areas to using formulas.

Evaluation

Have the students complete the Area Evaluation Worksheet, p. 59, **without** the rubber bands. (Answer: Students will create a figure with sides that are twice the length of the given figure, then find the area of the enlarged figure based on the area of the original.)

Journal assignment
Have students explain the relationship between the areas of figures that have doubled sides and the process involved.

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Extension activity

- Have students create 3-to-1 enlargements by adding a third rubber band and following the original object with the knot **farthest** from the pencil. Have them follow the original object with the knot **closest** to the pencil to produce a 3-to-2 enlargement.

Connections to other subjects

Art. Replicate a figure using the rubber band activity, then incorporate it into a design or offer students a tessellation activity.

Resources for teachers

Introduction to Tessellations by Dale Seymour and Jill Britton (Dale Seymour Publications)
Math by All Means: Area and Perimeter by Cheryl Rectanus (Cuisenaire Company)
Ask Dr. Math at www.oplin.lib.oh.us

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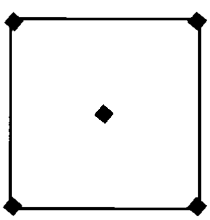
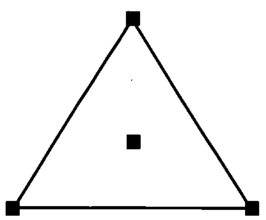
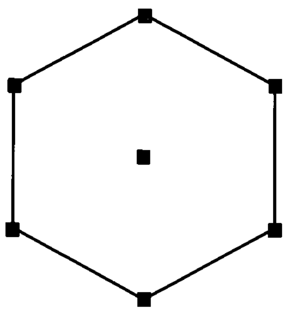
Enlargement Worksheet 1



Enlargement Worksheet 2



Enlargement Worksheet 3



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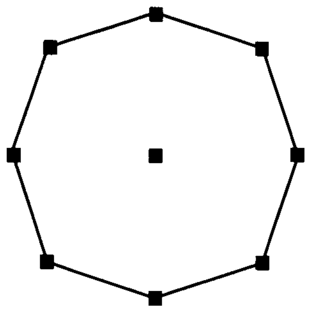
Enlargement Worksheet 4



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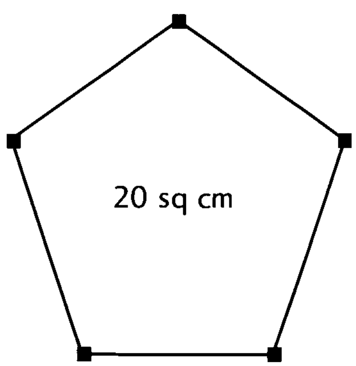
Area Evaluation Worksheet

Make a figure with sides 2 times as long as the figure you see here. You may use scissors and tape, but not rubber bands. The number inside the figure represents the area of the original figure. Find the area of the figure you make.



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Double Equals Quadruple

About this learning activity

Students will draw floor plans in different sizes and calculate how each room area increases as the room's dimensions increase.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I P	III ○	V ○	VII ○
II ○	IV X	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

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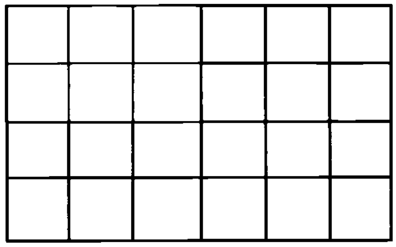
Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

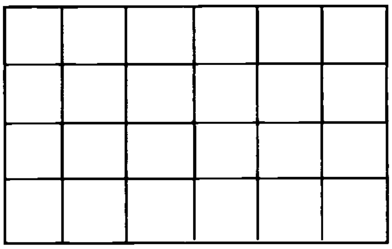
Materials

- Graph paper
- Overhead projector
- Graph paper on transparency
- Squares of tag board (1 for each student)
- Floor Tiles Chart worksheet

Engagement activity



Using an overhead projector, demonstrate coloring 2 blocks down and 3 blocks over (see illustration). Have them do the same on their graph paper.

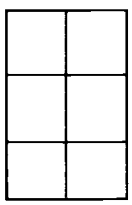


Next, have students double the blocks to 4 blocks down and 6 blocks over. Illustrate it on the overhead and let students check their work.

Help students discuss the differences in the sizes of the 2 patterns they drew. (Answer: Doubling the length of the sides increases the pattern area by 4 times the first pattern's area.)

Exploration activity

1. Explain to the class that Mary wants to put 1-foot tiles on the floor of a 2-foot by 3-foot rectangular room. Instruct students to determine how many tiles Mary needs by drawing her floor with the tiles in place on graph paper.



Mary's floor

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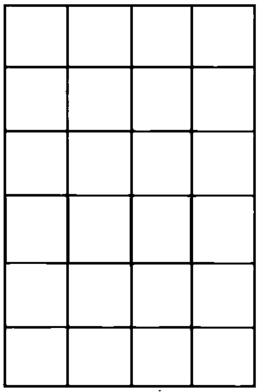
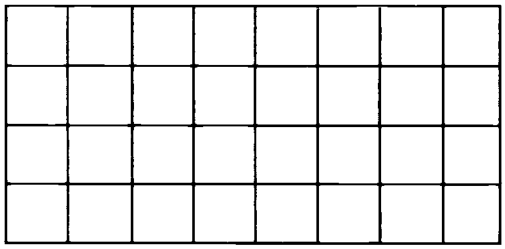
2. Next, explain to students that Jody likes Mary's floor so much that he wants to put the tiles on his floor and that Jody's floor is twice the size of Mary's. Have students determine how many tiles Jody will need by drawing his floor with the tiles in place on graph paper.



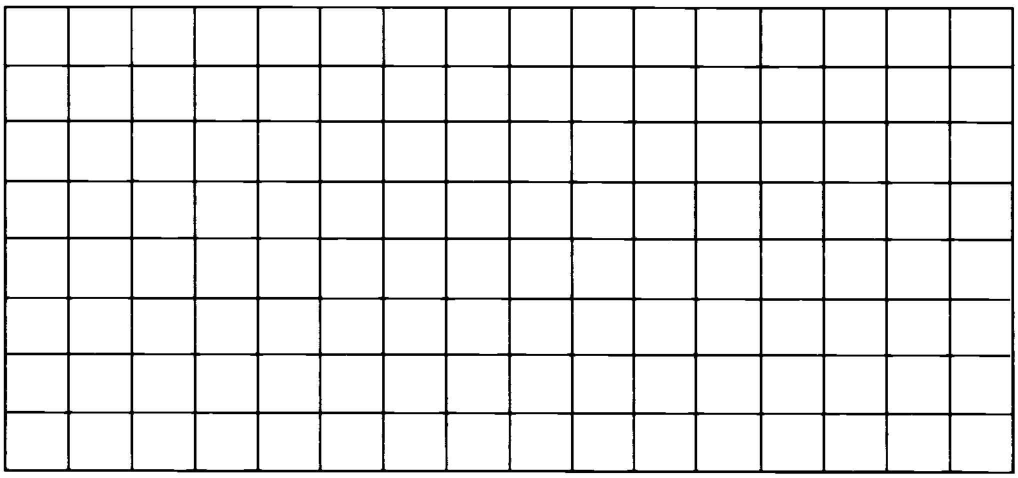
Jody's floor

3. Tell students that Jody's friend, Dianna, really likes her floor and wants to put the same tiles on her 4-foot by 8-foot floor. Have students determine how many tiles Dianna will need, using the same method as before.



Dianna's floor

4. Discuss with students that now Fred wants to do his floor in the same tiles and that his floor is twice the size of Dianna's. Instruct them to use the same method to determine how many tiles Fred will need.



Fred's floor

75

5. Have students compile the data from their graphs on the Floor Tiles Chart worksheet.

6. Facilitate a class discussion with these and other questions:

    ? How do you think the process we used to calculate the number of tiles needed for each room would work with rectangles? Triangles? Note: Have students use their tag boards to show what happens to rectangles and triangles.

    ? What do you think will happen with a parallelogram?

Real-life applications

Have students brainstorm all the jobs that might use this type of information (e.g., carpenters, landscape architects, general contractors, carpet installers, tile floor installers). Then, help them recognize ways that they will use similar information when they become homeowners.

Explanation

Doubling the sides of a figure will make the area 4 times larger. For example:

2 x 3 = 6	
4 x 6 = 24	(24 is 4 times the previous area of 6)
8 x 12 = 96	(96 is 4 times the previous area of 24)
16 x 24 = 384	(384 is 4 times the previous area of 96)

Evaluation

Provide a partially completed chart containing data that uses the *4 times rule* and ask students to complete the chart.

Journal assignment
Instruct students to describe the *4 times rule* and write an example in their journals.

Extension activity

☼ Have students use tag board cutouts to prove the *4 times rule* with different shapes. Then, discuss perimeter and area, and practice finding both for the figures that students cut out.

Home connections

Ask each student to measure the floor of his or her bedroom (or any room in the house) and draw it on graph paper with accurate dimensions.

Resources for teachers

When Are We Ever Gonna Have to Use This? by Hal Saunders (Dale Seymour Publishers, 1988)

Show What You Know—6th grade (Englefield & Arnold Publishers)

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Floor Tiles Chart

Student	Room Size	Number of Tiles Needed
Mary		
Jody		
Dianna		
Fred		

78

Double Exposure!

About this learning activity

Students will use a grid system to enlarge squares and a drawing.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I P	III X	V X	VII ○
II ○	IV X	VI X	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Color tiles (optional)
- $8^1/_2$ x 11-in sketch (See sample, p. 70, or choose another **simple** sketch.)
- 18 x 24-in drawing paper
- Rulers
- Scissors
- 1-in grid paper
- Markers and/or colored pencils

Hint: For this activity, you will provide a sketch on $8^1/_2$ x 11-in paper for students to dilate (stretch) onto an 18 x 24-in sheet of drawing paper. It is recommended that the sketch be put on 1-in grid paper before distributing it to students.

Engagement activity

Have students complete the Growing Squares activity, p. 69. If desired, let students use plastic color tiles to complete the activity.

Exploration activity

1. Distribute the sketch. Divide the number of 1 x 1-in squares in the sketch evenly among the students. Keep 1 square for yourself for demonstration purposes.

2. Double your square's size (creating a blank 2-in square). Ask students what "double sizing" the square does to its area.

3. Enlarge your 1-in section of the sketch from 1-in to the same size as the "double-sized" square. Note: Redraw your section into a 2-in square with colored transparency pens.

4. Instruct students to "double size" their portions of the sketch too, using markers and/or colored pencils. Encourage them to color the sketch pieces.

5. After all the 1-in pieces have been dilated, have students place their "double-sized" squares on an 18 x 24-in sheet of paper to create the original sketch (like working a puzzle).

6. Conduct a class discussion with these questions:

   - What is the area of 1 of the small squares? (Have students measure, if needed.)
   - What is the area of 1 of the "double-sized" squares?
   - How many small squares does it take to make 1 "double-sized" square?

? What is the ratio of the small square to the "double-sized" square?

? What is the area of the original sketch?

? What is the area of the enlarged sketch (i.e., the puzzle we put together)?

? How does this activity compare to the Growing Squares activity we completed earlier?

Evaluation

- Ask students to explain what happens when they double the dimensions of a square.
- Have students calculate the size of paper needed to double the "double-sized" square.

Journal assignment
Ask students to suppose they have a 2 x 3-in wallet size picture and want to double its size. Have them calculate the dimensions of the new picture and explain what doubling will do to the image (wallet size picture).

Extension activity

☼ Repeat the exploration activity, using isometric dot paper and dividing the picture into 1-in equilateral triangles.

Growing Squares

Using squares, continue the pattern shown in Figure 1 and complete Table 1. By looking for the pattern, find "n" and complete the values for the solutions.

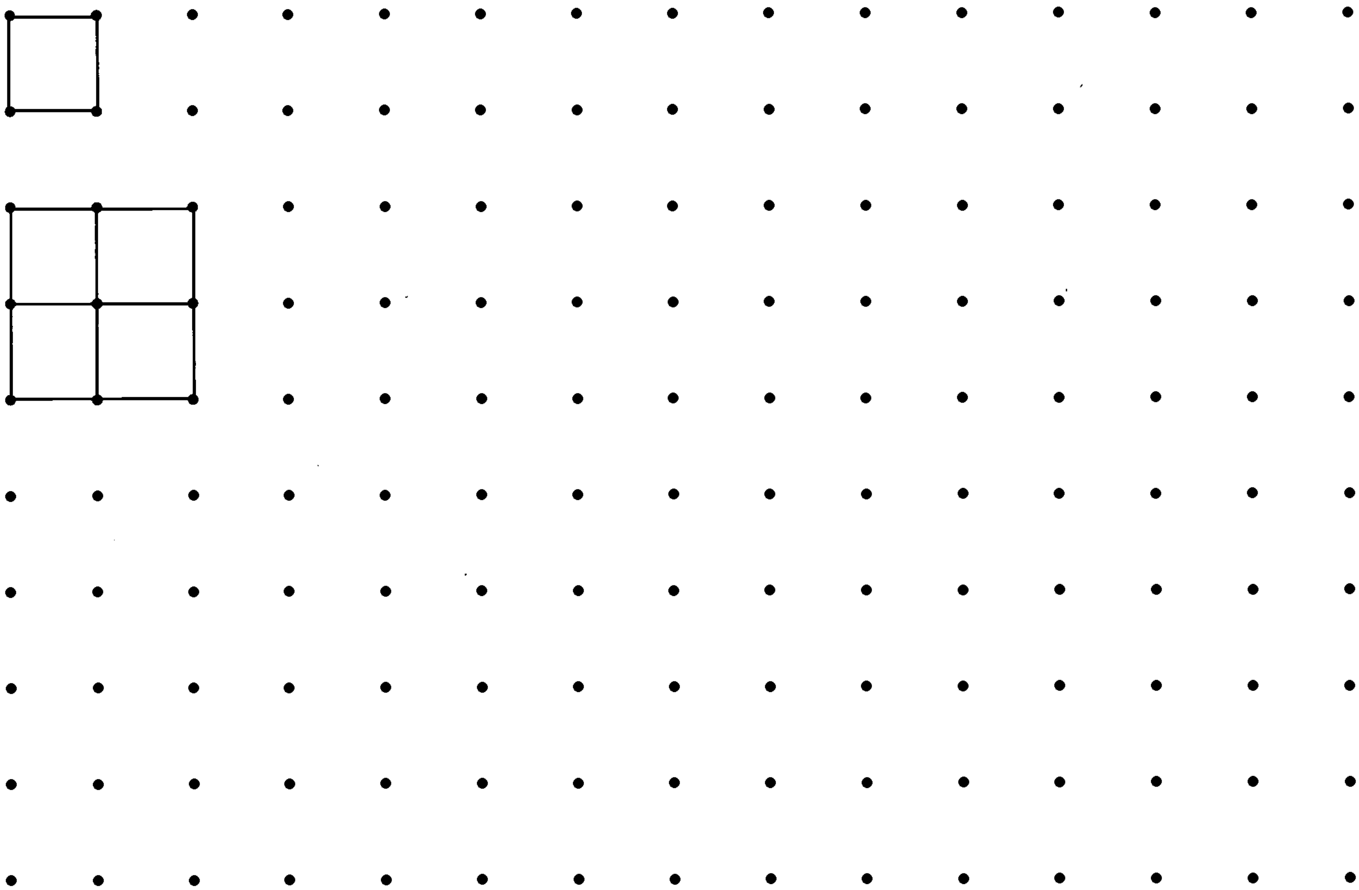


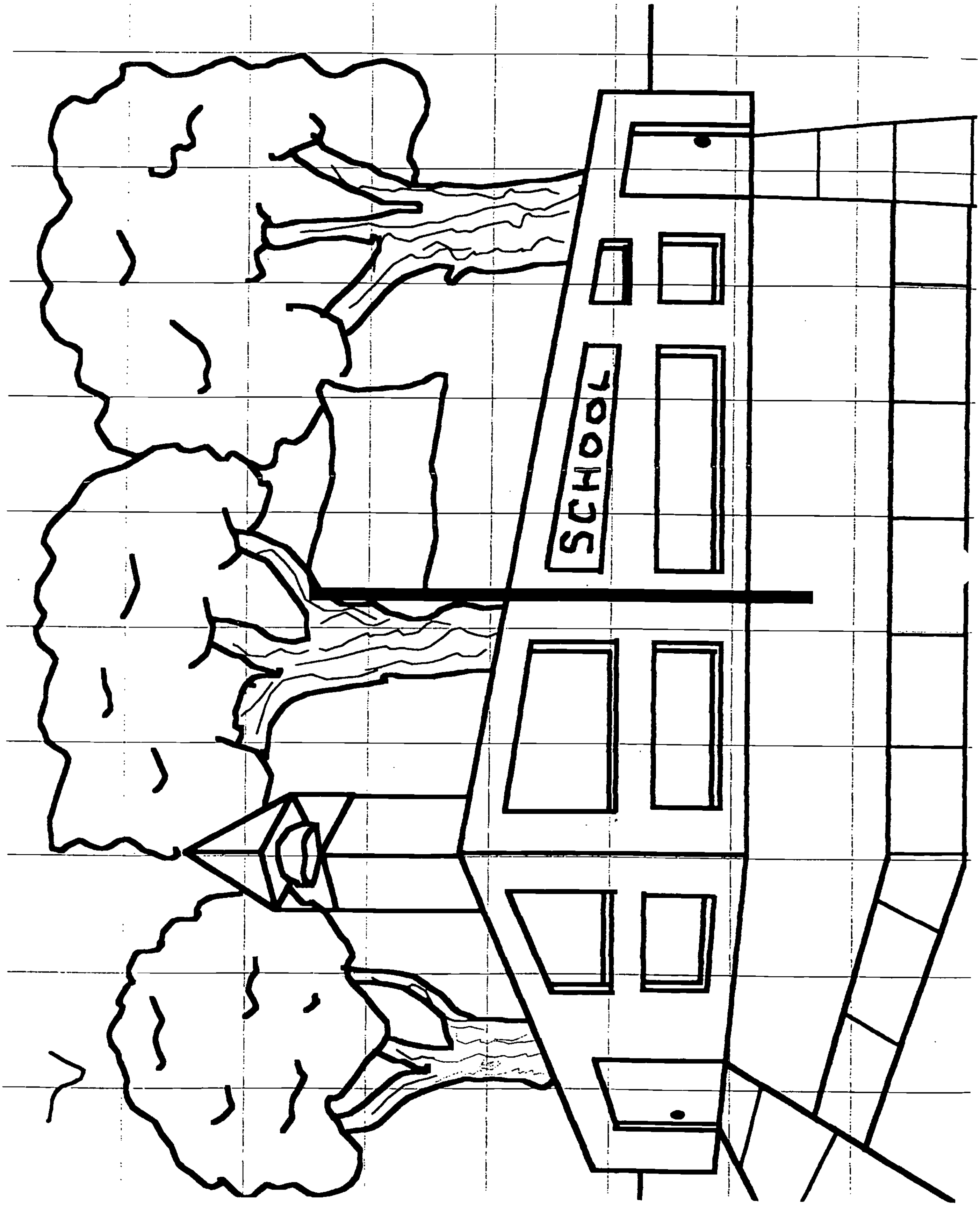
Figure 1

Table 1

Solutions

Squares	1	2	3	4	5	n
No. of squares added to make larger square						
Total number of squares used						
Perimeter						
Scaled edge (number of squares per side)	82					

Double Exposure! Sample Sketch



Will the Flowers Grow?

About this learning activity

Students will design flower beds for the school, then calculate changes that occur in the flower bed areas as their dimensions change.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I P	III ○	V ○	VII ○
II ○	IV X	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Colored tiles or construction-paper squares
- Grid paper

Engagement activity

Present to students the following scenario:

> *Our principal has asked us to design flower beds for our lawn area around the school. Use your grid paper and squares to make a map of our school grounds, then decide where to put the flower beds and what size they should be. The only controlled variable (rule) is the shape of the flower beds—all of them must be square.*

Allow the students to work in pairs. Review scale and direct students to construct their maps according to a scale decided upon by the class. Take students outside to make decisions about flower bed placements and sizes. Give students ample time to take measurements.

Exploration activity

1. After maps and designs have been completed for the engagement activity, review the concept of area with the students.
2. Have the students calculate their flower bed areas and record them on their designs.
3. Then, inform the students that the principal loves flowers and wants bigger flower beds. Instruct them to double the length of each side of their flower beds. Note: The students will need to rethink their maps and designs.
4. Once student pairs are satisfied with their new designs, have them draw and label their maps on the grid paper.
5. Ask students to calculate and record the areas of their new flower beds.
6. Have some pairs discuss how they arrived at their designs and how the designs had to change when they doubled the flower bed sides. Also ask them to explain how the areas of their flower beds changed when the sides were doubled.
7. Have several pairs put their data on the board. Help students conclude that **when the *sides* of a square are doubled, its *area* quadruples.**

Real-life applications

Help students identify situations in which knowledge of this math principle might be helpful, such as when working with contractors for home building, remodeling, or repair projects.

Evaluation

On a separate sheet of grid paper, have each student pair draw only its flower bed designs (before and after the sides were doubled). Have pairs include the dimensions and areas of each bed, and write a paragraph explaining what happened to the area when the sides were doubled.

Journal assignment

- Have students describe in their journals the mathematical thinking and processes they used to complete this activity.
- Instruct students to discuss jobs (e.g., designers, landscapers, builders) that might require these skills with their partners, then write about the jobs.

Extension activity

☼ Have students explore other figures (i.e., equilateral triangles and hexagons) to prove that the same (quadruple) relationship exists.

Connections to other subjects

Art. Discuss design and landscaping activities. Invite an expert to speak to the class.

Science. Have students conduct research about plants, then facilitate a class discussion with questions like:

? What will grow here?

? During what season(s) will each grow?

? Do we need fertilizer, mulch, or other materials?

? How much of each do we need and why?

Home connections

Have each student design a flower bed for his or her yard, then record the design and an estimated cost to plant it. Next, ask each student to determine and record the garden's area and cost after the size is doubled.

Resources for teachers

Everyday Math by Marge Lindskog (Frank Schaffer Publications, 1996) (p. 11)
Hundreds of Board Activities by Marilyn Preddy (Carson-Dellosa Publications, 1995) (p. 50)
Frog Math Power, Level D by Mary Jo Hand (Frog Publications, 1995) (game 24)
Area by J. Srivastava (Thomas Y. Crowell, 1974)

Strand II: Problem-Solving Strategies

Learning Activities **Page**

Notes

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Get the Picture?

About this learning activity

Students will apply 4 different problem-solving strategies (i.e., making a drawing, using a chart/table or graph, picking out key words, and developing a number sentence) to solving word problems.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II P	IV X	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

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Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Hint: Create a math problem-solving center for the exploration activity. You may wish to create enough materials to implement a single center or several centers at once.

Materials

- A word problem on an index card or poster (see engagement activity)
- Situation cards (see exploration activity)
- Solution cards (see exploration activity)

Engagement activity

Present the following problem to the class:

Your class is holding a car wash to raise funds for the annual trip to Washington, D.C. You charge $5.50 for each car and $7.00 for each truck. A local car dealer informs you that the dealership will donate $1,000.00 for your trip if you wash 30 trucks. You earned $537.50 during your fund-raiser, washing 89 vehicles. Do you get the $1,000.00 donation?

Challenge students to find the solution by creating a chart. (Answer: Yes. See the solution on p. 81.)

Hint: This is written as a cooperative group activity, but can be adapted to fit your needs.

Exploration activity

1. Copy and cut the situation and solution cards for a matching game. (See p. 80.)

2. Have students work in small groups to find the **best** solution method. Instruct them to match a solution strategy to each problem, then solve each problem. (Solutions can be found on p. 82.)

3. When students have been given ample time to complete the problems, discuss the situation cards and their solutions. Then, ask the following questions:

   ? What signals or structures contained in word problems help you to recognize the best solution method for a problem?

   ? Is there more than 1 way to solve a problem?

   ? Do you think situations such as those we studied today happen in the real world? Give examples.

Evaluation

Distribute Mission Mathematics problems for students to solve. (See instructions and sample on p. 83.) Have students solve the problems in small groups or individually, using the strategies studied.

Hint: You can have students write number sequences and have classmates solve them.

Journal assignment
Have students create a family tree that goes back at least 3 generations, list the ethnic roots of each parent and grandparent, and try to calculate the percentage of each ethnic heritage they carry in their genes.

Extension activity

- Invite representatives from local businesses to the classroom to share their use of math in daily problem-solving situations. Ask them to present problems they frequently face at work. Challenge the class to attempt to solve them, using the strategies they have studied.

Connections to other subjects

Social Studies. Let students practice map skills by creating a street route for a new mail carrier and drawing a map of the route.

Language Arts. Have students describe in words only the new mail route created in the Social Studies activity.

Home connections

Have students ask a family member to help them design a yard sprinkler system for their front yards. Instruct them to diagram the system on paper and calculate the amounts of pipe, fittings, and other materials that will be needed. Have them contact a local hardware store to secure pricing information and calculate a reasonable estimate of the material costs.

Resources for teachers

Appetizers and Lessons for Math and Reason (a collection of problem solving and mathematical reasoning from NCTM 5–8) Web site address: http://www.cam.org/aselby/lesson.html
Ask Dr. Math at www.yahooligans.com

Exploration Activity Cards

Situation Card A

The Computer Club will meet on Tuesdays and Thursdays during the month of March. If their first March meeting is held on Tuesday, March 3, how many times will they meet in the month of March?

Situation Card B

How many different ways can you make 27¢ using only pennies, nickels, and dimes—*and* use all 3 coins?

Situation Card C

A truck carried 1589 lbs of cargo. Another carried 3280 lbs. How much more than 2 tons did the cargo trucks carry?

Situation Card D

At the Metal Products R Us plant, the shipping department has a priority list. Bolts are more important than springs, which are more important than clips but not as important as cotter pins. If cotter pins, springs, and bolts show up on the shipping dock together, in what order will they be shipped?

Solution Card

Make a drawing.

Solution Card

Use a number sentence.

Solution Card

Use a graph or table/chart.

Solution Card

Use words.

Engagement Activity Answers

1. The information we know:

   - Cars = $5.50 each
   - Trucks = $7.00 each
   - Earnings = $537.50*
   - Total washed = 89 (from adding trucks and cars)

**Total income from the cars and trucks. We know the .50 indicates both cars and trucks were washed.*

2. Chart should track:

   - Number of cars washed
   - Number of trucks washed
   - A tally of total vehicles washed
   - Total earnings

   Since we are hoping to wash 30 trucks, put 28, 29, 30, 31, and 32 in the truck row to get started. Create combinations of 89 by calculating the correct number of **cars** (i.e., subtract total trucks in each column from 89). Place the **car** totals in the appropriate boxes in the Number of Cars row. Compute the **vehicle** totals. Answer: 32 trucks, 57 cars. Yes, our fund-raiser gets the bonus!

No. of Cars/Earnings	61/$335.50	60/$330.00	59/$324.50	58/$319.00	57/$313.50
No. of Trucks/Earnings	28/$196.00	29/$203.00	30/$210.00	31/$217.00	32/$224.00
Total Vehicles/Earnings	89/$531.50	89/$533.00	89/$534.50	89/$536.00	89/$537.50
Outcome *(Goal: $537.50)*	No	No	No	No	Yes

Results: Washed 57 cars at $5.50 for $313.50
Washed 32 trucks at $7.00 for $224.00
Totals: 89 vehicles $537.50

We washed over 30 trucks, so we get the bonus!

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Exploration Activity Answers

Situation A: Make a drawing to solve the Computer Club problem.

Sun.	Mon.	Tues.	Weds.	Thurs	Fri.	Sat.
			3		5	
			10		12	
			17		19	
			24		26	
			31			

Solution: 9 meetings

Situation B: Make a table/chart to solve the 27¢ problem.

1¢	2	2	7
5¢	1	2	2
10¢	2	1	1
Totals	27/27	27/27	27/27

Situation C: Use a number sentence to solve the cargo truck problem.

1,589 lbs + 3,280 lbs = 4,869 lbs
4,869 lbs – 4,000 lbs = 869 lbs over 2 tons

Situation D: Use words to solve the metal products problem.

Substitute *which* with *springs.*

Shipping Order:

1. Bolts (more important than springs)
2. Cotter Pins (because springs are not as important)
3. Springs (more important than clips)

Mission Mathematics

Instructions

Create a story problem like those used in the exploration activity. Put it on an index card. Deliver the card in a small brown envelope with a Mission Impossible-style message about the importance of solving it in a hurry. Note: Problems should be tailored to the needs and abilities of your students.

Hint:
This is a good opportunity for reviewing problems from your text or old math tests.

Sample

Secret Mathematician ______________:

Your mission is to help Slick Sam's Bath Mat Company. The company secretary, Mr. Stumble, has to prepare the books for an Internal Revenue Service audit. He forgot to keep track of postage expenses. He can recreate them by figuring out how many flyers were mailed at 25¢ each, the number of sample materials mailed at $1.30 each, and how many coupon sheets were mailed at 45¢ each. The company spent a total of $83.50 on postage and mailed 105 items.

Time is critical! This message won't self-destruct and if you throw it away, agent ______________ (disguised as your teacher) will stay on your case until you find a solution. What are you waiting for? **Get busy!**

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Who's Afraid of Word Problems?

About this learning activity

Students will create word problems, then solve them by converting to number sentences.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V X	VII ○
II P	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

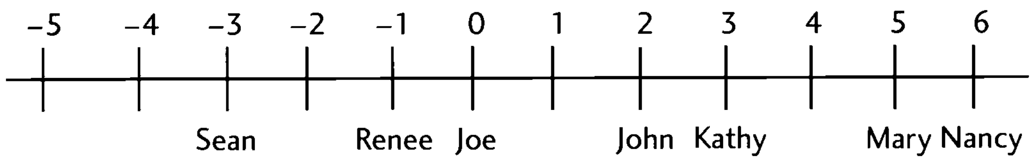
Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Number lines
- Paper and pencils

Engagement activity

1. Draw a large number line on the chalkboard. Have students stand at various places on the number line (see illustration).



2. Ask students various questions about the places on the number line. For example: Who is located at 3? Help students notice that 2 people are located at 3—Sean at −3 and Kathy at +3.

3. Discuss the direction of the number line. For example: If Mary moves over to stand with John, how many spaces does she move and in which direction? If Mary moves to join Nancy, how many spaces does she move and in which direction?

4. Show the students how they can write a number sentence for the preceding examples.

   - Mary moves 3 in a negative direction to meet John. $5 + (-3) = 2$
   - How many spaces does Mary move to meet Nancy? $5 + ? = 6$

Exploration activity

1. Distribute number lines (or have students make them). Have students use their number lines to make a number sentence for the following problem:

   Your football team gained 3 yards, then lost 2 yards. How many total yards were gained?

2. Have students work in groups of 2 or 3. Instruct them to write several additional examples of yards gained and lost to share with another group.

3. Once students are comfortable using small numbers, introduce a variety of more difficult problems.

    - Jesse sold 200 coupon books for the DECA Club. He was given 350 to sell. How many did he have left to sell? (Answer: 350 – 200 = ____)

    - Mary sold a gently used prom dress at the school store's annual Prom Dress Resale. She paid $35 for the dress when she bought it. She sold it in the school store for $18. How much did Mary lose on the dress? (Answer: $35 – $18 = ____)

5. When students are comfortable using addition and subtraction, introduce multiplication and division word problems. Example: Tell students that you are thinking of a number that is 5 times the number plus 4. Write a number sentence on the board: ? x 5 + 4 = ____.

6. Have students make up additional "I'm thinking of..." problems to share with the class.

7. Through discussion about word problems and number sentences, help students recognize that word problems contain key words that can help them identify the mathematical operations necessary to solve them.

Real-life applications

Help students recognize that we solve word problems every time we go to the store. Examples:

- Milk is $1.98 per carton. If I buy 2 cartons, how much money will I need?

- If I buy pears for $1.29, apples for $2.09, and strawberries for $3.78; how much change will I get back from a $10.00 bill?

Explanation

Most students struggle with solving word problems. By creating word problems, students will begin to feel more comfortable solving them.

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Evaluation

Have each student write 4 word problems and the corresponding number sentences.

Journal assignment
Ask students to explain how a word problem can be translated into a number sentence and to provide an example.

Extension activity

- Instruct students to create word problems from newspaper advertisements, then use them to create a classroom bulletin board. Challenge another class to solve the problems.

Connections to other subjects

Art. Have students complete an art-related word problem. Example: If I mix 2 cups of blue paint and 3 cups of yellow paint, how many cups of paint will I have?

Music. A song is designed to be sung in 14 minutes, but the choir sings it in 8 minutes. How much faster than intended did the choir sing it?

Home connections

Have students go grocery shopping with a family member and record the number of times they can solve a word problem.

Resources for teachers

When Are We Ever Gonna Have to Use This? by Hal Saunders (Dale Seymour Publications)
Mathercise by Michael Serra (Key Curriculum Press, 1992)
Super Solvers Outnumbered! by The Learning Company (computer program)
Math Blaster Mystery by Davidson (computer program)

Where's the Beef?

About this learning activity

Students will practice identifying necessary, unnecessary, and omitted information and relate this skill to solving math problems.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II P	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

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Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Empty detergent containers in various sizes (both powder and liquid)
- Construction paper price tags (affix to the detergent containers)
- Circulars from various retailers (enough for a research exercise)
- Product Fact Sheets

Engagement activity

Display the detergent containers with the price tags affixed to them. Ask students to find the best detergent bargain.

Exploration activity

1. Display/distribute the store circulars.

2. Let students draw or choose items (e.g., paper towels, fruit, cleaning agents) from the circulars to research.

3. Have them record all the information they can find for their products on their fact sheets (see the sample on p. 91). Instruct them to include 3 prices—1 close, 1 exact, and 1 unreasonable.

4. Organize students into pairs and instruct them to trade fact sheets. Tell them to examine each other's data, record any questions left unanswered, and mark any information they find unimportant. Finally, ask students to choose a price for the product based on the information given.

5. Call on students to report the results of the activity.

6. Facilitate a class discussion with these and other questions:

   ? What general categories of important mathematical information did you find? (Possible answers: Price, quantity, weight, volume.)

   ? What general categories were unimportant? (Possible answers: Color of package, name brand, expiration date.)

Explanation

Finding the right information can be a challenge for young mathematicians. Help students recognize through these hands-on exercises how getting the **right** information, and **all** the necessary information, is important to solving a problem.

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Evaluation

Use a box knife to "undo" a box that will flatten to fit on an 8½ by 11-in sheet of paper. Copy the box. Ask students to circle all **important information in red** and **unimportant information in blue**. Also have them record in pencil any information that was needed but not provided.

Journal assignment

Have students compare the process of a court trial to solving a math problem. *(Hint: If necessary, help students draw correlations between the 2 processes prior to making this assignment. Help them recognize that in a court trial participants present information that is needed and information that is not needed, and that some information is left out by accident or purposefully. Then, explain that in math some information is needed and clear, some is left out, and some is not needed.)*

Extension activity

- Videotape some commercials. Have students analyze the commercials to identify the information provided, what was left out, and any unnecessary information. Challenge students to find information that would help them to determine whether a product shown is a good buy. Have them report their findings to the class at a later time.

Connections to other subjects

Language Arts. Analyze newspaper stories for facts that are given, not given, or unnecessary. Evaluate the reporter's work on the merits of the factual information.

Home connections

Have students watch the news and some commercials with a family member, then discuss the information presented, e.g. what information was given, needed, and irrelevant to the story or commercial viewed.

Resources for teachers

Problem Solving Activities, The Intermediate Mailbox (June/July 1993–1994) See pp. 25–31.
Problem Solving Strategies, Education Center Inc. (1996–97) See pp. 36–41.
Ask Dr. Math at www.yahooligans.com

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Product Fact Sheet

Student ____________________ Partner ____________________

Product ____________________

Facts:

Suggested retail price: ________ ________ ________

Comments: ____________________

Chosen price: ________

Reason: ____________________

Teacher comments:

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What Do I Need?

Hint: This activity uses skills developed in the preceding Strand II activities. It should be implemented ***after*** *students have acquired some basic problem-solving skills.*

About this learning activity

Students will select given, needed, and irrelevant information as they create and solve word problems.

Process skills

○ Building models	○ Interpreting data
○ Categorizing or classifying	○ Measuring
○ Communicating	○ Observing
○ Comparing	○ Ordering
○ Controlling variables	○ Predicting
○ Experimenting	○ Reasoning
○ Hypothesizing	○ Recognizing relationships
○ Inferring	○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II P	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Poster "cards" (engagement activity) Note: Prepare poster "cards" in advance, putting 1 piece of the problem information (see list) on each poster:
  - Mary
  - John
  - Picnic
  - At T.J. Evans
  - 43 people
  - Want hamburgers and hot dogs
  - Decide only hamburgers
  - 2 hamburgers per person
  - How many hamburgers do they need to buy?
- Poster boards (for exploration activity)
- Markers
- Information Categories chart (3–4 copies per student)

Hint: You may wish to use flip chart paper instead of poster board.

Engagement activity

1. Select 9 students and have them hold the poster "cards" in front of the class. Read the following word problem aloud as the students line up.

   Mary and John want to have a picnic at T.J. Evans Park. They plan to invite 43 people. Mary wants to serve hamburgers and hot dogs, but they decide they will only serve hamburgers. They plan to serve 2 hamburgers per person. How many hamburgers do they need to buy?

2. Explain to the class that the first step in solving a word problem is to list the information given and that a chart is a helpful way to approach this step. Distribute an Information Categories chart to each student, with instructions to list all information given.

3. Ask the class to eliminate the information they do not need (i.e., ask the students holding the unnecessary information to sit down, have students list the irrelevant items in the third column of their charts).

4. Have students name any additional information they need to solve the problem and add it to the first column.

5. Now change the problem by substituting, "They plan to invite a few friends" for "43 people" and repeat steps 2 through 4.

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Exploration activity

1. Divide students into groups of 4–6 each. Have each group create a story problem, following the format used in the engagement activity.

2. Encourage students to add irrelevant items and/or leave out some information that is needed to solve the problem.

3. Have groups put their stories on poster "cards" and present them to the class.

4. Instruct the groups to exchange stories and complete Information Categories charts. Have them try to solve the problems, if desired.

Real-life applications

Help students recognize that the ability to identify and sort word problem information can be useful throughout their lives. Provide examples like those given, then help students identify others.

Do I have enough information to make a wise decision about:

- Choosing a college or tech school.
- Buying a car.
- Going to the movies.

Explanation

Students need to learn to identify and sort word problem information into these categories: given, needed, and irrelevant. Help students recognize that this skill is essential to solving problems.

Evaluation

Provide several problems and ask students to complete a chart that lists the given, needed, and irrelevant information for each problem. Note: Students should answer "yes" or "no" to the question, "Do I have enough information to solve this problem?" (last column in the Information Categories chart, p. 96).

Journal assignment
Have students complete this sentence, "Before making decisions, I must have the following information...."

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Extension activity

☼ Distribute copies of movie advertisements from the local newspaper. Note: Be sure that the **show times are not listed**. Ask students to select a movie and list all of the information that they need to have before attending the movie. Examples:

- Time the movie begins and ends.
- Where is the theater?
- Can I get a ride there and back?
- Do I have enough money to purchase a ticket?

Connections to other subjects

Art and Science. Ask art teachers and science teachers to assign projects that require students to begin by identifying given, needed, and irrelevant information.

Language Arts. Have students write stories that contain both relevant and irrelevant information. Then, ask students to read their stories to the class and have classmates identify the irrelevant information in the stories.

Home connections

Have students discuss with family members how this skill (i.e., identifying needed, relevant, and irrelevant information) can be used in the home. Then, instruct students to combine their responses, making a list to display in the classroom.

Resources for teachers

When Are We Ever Gonna Have to Use This? by Hal Saunders (Dale Seymour Publications)
Mathercise by Michael Serra (Key Curriculum Press, 1992)
Everyday Math (computer program)
Super Solvers Outnumbered! by The Learning Company (computer program)
Math Blaster Mystery by Davidson (computer program)

Information Categories

Name(s) ____

Needed Information	Given Information	Irrelevant Information	Can I solve this problem with the information I have?

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Robot Logic

About this learning activity

Students will use only verbal instructions to direct a "robot" to complete a task.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II P	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

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Materials

- Paper and pencils
- Padded walls

Engagement activity

This learning activity does not require an engagement activity.

Hint: You may wish to conduct this exercise in a room with padded walls.

Exploration activity

1. Choose a simple task like sharpening a pencil or retrieving an object from a table. Decide what directions you want to be preprogrammed into the "robot" (e.g., right, left, stop, walk, up, down). Note: You want to provide the basics, but not too much knowledge.

2. Choose 2 student volunteers—a "robot" and a recorder. Explain their duties (see Explanation section). Be sure to stress to the "robot" that he or she must carry out the class' directions **exactly**.

3. Explain the task and preprogramming to the class. Instruct students to discuss the directions they will give and let the recorder write them on flip chart paper. Encourage students to be specific, giving an example like, "If the class says walk, the "robot" walks until you say stop."

4. Then, have students instruct the "robot" verbally, using the directions decided on by the class and giving only 1 command at a time.

5. Instruct students to continue giving directions until the task has been completed.

6. Help students apply this logic format to any problem-solving situation by having them write a problem in words before working out the solution in mathematical terms. Note: This strategy teaches students to concentrate on the problem situation rather than the numbers when they first read a problem.

7. After students have recorded the problem in word format, let them translate their words into math terms and solve the problem.

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Explanation

Expect these "robot" tasks to take a long time. Students need lots of practice to direct the "robot" correctly. (One class took the entire school year to have a "robot" tie his shoelace.) Let the class work on it over several days, weeks, or even months—whenever you have a few minutes between activities. Keeping the recorder's flip chart sheet of directions posted in the classroom will make it easy to switch to the "robot" task whenever it is convenient.

The "robot" has the most important job. Choose a student who likes being the center of attention and will do anything, including looking foolish, to make everyone laugh. The "robot's" job is to do **exactly** what he is told—nothing more and nothing less.

Remember to keep the tasks simple. And avoid messy tasks like making chocolate milk.

Connections to other subjects

Computer Science. Show students the coded language of a simple computer program. Help them recognize that the step-by-step process is what the computer reads and carries out. Talk about the number of steps needed to perform even the simplest tasks, then speculate about the process and steps required to program an application.

Language Arts. Have students write a process paragraph—write directions for completing a task (e.g., making a peanut butter sandwich, finding the average of a set of numbers). Older students can apply this logical order to writing essays and term papers (called *the flow of the essay*).

Home connections

Ask family members to talk through their logic aloud as they teach a process at home. Explain that the students need to hear the order, understand the reasoning, and remember the process.

Math Works in the Real World

About this learning activity

Students will practice various problem-solving strategies in completing 3 sets of activities.

Process skills

○ Building models
○ Categorizing or classifying
○ Communicating
○ Comparing
○ Controlling variables
○ Experimenting
○ Hypothesizing
○ Inferring
○ Interpreting data
○ Measuring
○ Observing
○ Ordering
○ Predicting
○ Reasoning
○ Recognizing relationships
○ Recording

Ohio mathematics proficiency strand(s)

I	○	III	○	V	○	VII	○
II	P	IV	○	VI	○	VIII	○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

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Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Varsity Tennis Squad problem on poster board (see engagement activity)
- 15 5 x 7-in manila envelopes
- Large bag of peppermint candies
- Instruction card (see Explanation section, Station I)
- Stations Worksheet
- Cyberpet Production Report (on poster board)

Engagement activity

Display the Varsity Tennis Squad problem. Ask students, "How could we solve this problem?"

> *Karla wants to make the varsity tennis squad. She practices serving by hitting 10 serves each day for the first week. Then, she decides to increase her practice. On Monday of the second week, she hits 15 serves, then increases the number of serves she hits by 5 each morning. How many serves will Karla hit on Thursday morning of the second week?*

Exploration activity

The exploration activity is designed for students to work in small groups and move from station to station. You may need to adapt the learning methods to meet your students' needs.

1. Prepare the materials prior to implementing the exploration activity.

   Station I.

   a. Write the numbers *1* through *5* on 5 of the manila envelopes. Put 3 peppermint candies in the first envelope, 5 in the second, 7 in the third, 9 in the fourth, and 11 in the fifth.

   b. Label the second set of 5 envelopes *A* through *E* and place 1 candy in A, 2 in B, 4 in C, 8 in D, and 16 in E.

   c. Label the third set of envelopes *triangle*, *square*, *pentagon*, *hexagon*, and *heptagon*, then place 2 candies, 5 candies, 3 candies, 6 candies, and 4 candies in them, respectively.

   d. Place the groups of envelopes on a table with a card that instructs students not to change the contents of the envelopes or mix up the sets of envelopes. (Envelopes are labeled to avoid confusing them within the station.)

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Station II. Prepare a poster board chart of the Cyberpet Hourly Production Report (see p. 108). Post it in the Station II area.

Station III. You may wish to put Freddy's sales form on a poster board and post it in Station III (see Stations Worksheet, Station III).

2. Organize students into small teams. Distribute a Stations Worksheet to each student.

3. Instruct students to explore each station and answer the questions on the worksheet for that station.

4. Facilitate a class discussion with these and other questions:

   - How did you discover the patterns in the envelopes?
   - Were they obvious or did you have to work to identify them? Explain.
   - Where did you look to find your answers to the questions in Station II?
   - Were the answers obvious, or did you have to work to find them? Explain.
   - Why is the math work required for Station II important to a business owner?
   - If you worked at the Cyberpet Factory, how could you use this data?
   - What did you learn about doing math work from visiting Station III? (Answer: Always check your work!)
   - How can math affect your income? Your profits as a business owner?

Explanation

Station I Worksheet Solutions.

What did you discover about each set of envelopes? Be specific.

Answer: They contain patterns.

- *Envelopes 1–5 are a pattern (3, 5, 7, 9, 11) that increases by 2.*
- *Envelopes A–E illustrate a doubling pattern—2, 4, 8, 16, and so forth.*
- *The pattern in the third set of envelopes adds 3 to every other number (i.e., first, third, fifth and so forth), and subtracts 2 from the numbers in between (e.g., second, fourth).*

How many candies would be in each additional envelope if you extended the pattern by 3 more envelopes in each set?

Answers:

- *Set 1–5: Envelopes 6-8 would contain 13,15, and 17 candies, respectively.*
- *Set A–E: Envelopes F–H would contain 64, 128, and 256 candies, respectively.*
- *The next 3 shapes envelopes would contain 7, 5, and 8 candies, respectively.*

Station II Worksheet Solutions.

1. Anne; 74 units (Add horizontally.)
2. Sally; 11 units (Observe highest hourly amount.)
3. Anne; 9.25 pets per hour (Add each worker's total across the 8 hours and divide by 8.)
4. Bob; 1 unit (Observe lowest hourly amount.)
5. Bob [He has the lowest output of any worker and the lowest hourly production rate (1), and is 3 under the next closest producer.]
6. 7 per hour (Add the averages of each employee and divide by 7.)
7. Hour 8 is worst with 24 units; hour 3 is best with 45 units. (Add production tallies vertically.)

Evaluation

Observe and note students' progress as they work in the stations. Check their answers during the class discussion.

Journal assignment
Have students summarize what they discovered from each station.

Extension activity

- Help students videotape a "documentary" about validation and generalization of solutions to problems.

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Connections to other subjects

Social Studies. Give students local census data. Have them use the data to answer basic questions. Then, ask them to predict future trends based on the current data.

Home connections

Instruct students to look for patterns at home (e.g., quilts, floors, walls, ceilings) and sketch them or describe them for sharing with the class.

Resources for teachers

Patchwork Math Two by Debra Baycura (Scholastic Books, 1996)
Everyday Math by Marge Lindskog (Frank Shaffer Publications, 1996)
Homework Helpers (Frank Shaffer Publications, 1996) See p. 16.

Stations Worksheet

Name ___________________________________

Station I:

Explore the envelopes and answer the following questions.

What did you discover about each set of envelopes? Be specific.

How many candies would be in each additional envelope if you extended the pattern by 3 more envelopes in each set?

Station II:

Review the Cyberpet Hourly Production Report and answer the following questions.

1. Who will get the day's bonus for the best output?
2. Who has the highest output for any 1 hour period?
3. Who has the best average hourly output?
4. Who has the lowest output for any 1 hour period?
5. Who should get a "pink slip" for poor job performance?
6. What is the overall average hourly production for all the workers?
7. What is the worst hour for production? The best?

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Station III:

You are a teacher. One of your students, Freddy, runs up to your desk and declares that he has won the top sales award for your class fund-raiser. Is he right? Check his sales form to find the answer. Find and correct any errors.

Freddy's Sales Form

Icky Sticky Fund-raiser Company

"If it's tacky and cheap, we sell it!"

	Customer Name	Phone Number	Cheese on a Rope @$6.95	Bugs in a Mug @$3.95	Gummy Candy @$2.50	Plastic Light Pull @$7.95	Customer Total
1	John	555-1234	34.75	0	10.00	23.85	68.60
2	Cherie	555-6142	6.95	11.85	22.50	15.90	47.20
3	Juanita	555-4153	41.70	11.85	7.50	63.60	124.62
4	Eileen	555-4444	41.70	7.90	2.50	23.95	75.95
5	Samantha	555-6161	34.75	6.90	7.50	15.90	66.05
Total items sold:			20	10	16	18	67
Total sales:			$159.85	$39.50	$40.00	$143.10	$382.45

Freddy's Sales Form Answers

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Icky Sticky Fund-raiser Company

"If it's tacky and cheap, we sell it!"

	Customer Name	Phone Number	Cheese on a Rope @$6.95	Bugs in a Mug @$3.95	Gummy Candy @$2.50	Plastic Light Pull @$7.95	Customer Total
1	John	555-1234	34.75	0	10.00	23.85	68.60
2	Cherie	555-6142	6.95	11.85	22.50	15.90	47.20
3	Juanita	555-4153	41.70	11.85	7.50	63.60	124.62
4	Eileen	555-4444	41.70	7.90	2.50	23.95	75.95
5	Samantha	555-6161	34.75	6.90	7.50	15.90	66.05
Total items sold:			20	10	16	18	67
Total sales:			$159.85	$39.50	$40.00	$143.10	$382.45

Important: All values are the same on the student worksheet, except those that are written in **bold**.

Cyberpet Hourly Production Report

Employee	Hour 1	Hour 2	Hour 3	Hour 4	Lunch	Hour 5	Hour 6	Hour 7	Hour 8
John	7	9	10	8	0	6	8	8	6
Sally	8	10	11	10	0	9	7	5	5
Bob	4	6	6	5	0	3	4	2	1
Anne	9	9	9	10	0	9	9	9	10
Bud	7	10	9	6	0	7	5	3	2

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Strand III: Numbers and Number Relationships

Learning Activities **Page**

Notes

Multiplying to Get Less

About this learning activity

Students will experiment with candy and paper to discover that multiplying with fractions produces a smaller number.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III P	V ○	VII ○
II ○	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

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Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Small, individually-wrapped candies (20 per student pair)
- Sandwich baggies (4 per student pair)
- 9 x 9-in sheets of paper (1 per student)
- Rulers (inch)
- Crayons
- Multiplying Fractions worksheet (for evaluation)

Engagement activity

Give each pair of students 20 candies and 4 baggies. Explain to students that they may eat the candy after they complete the lesson. Instruct student pairs to divide the candy evenly, then divide each half of the candy again so that each student will have an equal amount of treats for today and tomorrow. When students finish dividing the candies, have them store the candy in the baggies and record (i.e., describe in writing) what they did in this lesson.

Exploration activity

1. Give each student a square of paper.

2. Ask students to use the rulers to make marks at the 3-in and 6-in points on their papers, then draw lines to divide the paper into thirds.

3. Have students fold their papers along the lines they drew, then unfold them and use red crayons to shade $1/3$ of the papers.

4. Instruct students to fold their papers in half, then unfold with the **colored side** facing up. Have them use blue crayons to shade $1/2$ of the papers, pointing out that they will be coloring over part of their red sections.

5. Ask students to look at their papers and name the fraction that represents the red portion. Then, ask them to identify the "blue fraction."

6. Ask students to identify the "red and blue fraction." Put the equation $1/2 \times 1/3 = 1/6$ on the board or overhead projector. Explain that first each student shaded $1/3$ of the paper red, then $1/2$ of the paper blue. Further explain that by multiplying these 2 fractions, we find that the section of each paper that is shaded **both** red and blue makes up $1/6$ of the paper—the combined red and blue section represents $1/3$ **of** $1/2$ of the paper.

7. Now help students apply this concept to the candy lesson (i.e., engagement activity). Ask student volunteers to describe how they completed the candy activity for the class.

8. Then, explain that first students separated $1/2$ of the total number of candies so that each member of the pair would get an equal amount of candy. Further explain that when they had to divide the candy in half again so each would have the same amount of candy today and tomorrow, they were calculating $1/2$ **of** $1/2$. Put the equation $1/2 \times 1/2 = 1/4$ on the board as you explain the exercise in these numerical terms.

9. Next, point out that after completing their first task, the team members had 2 equal portions (i.e., each had $1/2$ of the total) and that after doing the second task (i.e., dividing for 2 days), they had 4 equal portions, with each portion representing $1/4$ of the total. As you explain, illustrate the concept like this:

Student #1		Student #2	
10 candies ($1/2$ of the total)		10 candies ($1/2$ of the total)	
5 for today ($1/4$ of total)	5 for tomorrow ($1/4$ of total)	5 for today ($1/4$ of total)	5 for tomorrow ($1/4$ of total)

Equation: $1/2 \times 1/2 = 1/4$

10. Next, draw a square and demonstrate $2/3$ of $3/4$. Then, write the equation for the square: $2/3 \times 3/4 = 6/12$.

11. Direct students to look carefully at all of the equations written on the board. Facilitate a discussion about how to multiply fractions with questions like:

    ? Can you recognize any math principles to follow when multiplying fractions? (Answer: Multiply the numerators, then multiply the denominators.)

    ? Does this work for all fractions? (Note: Let students use the equations on the board to check.)

    ? What do you notice about the answers to all of these equations? (Answer: They are all smaller than the 2 fractions being multiplied.)

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Evaluation

Have students complete the Multiplying Fractions worksheet (p. 115) while eating their candies.

Journal assignment
Have students describe the exercises completed and the process of multiplying fractions. Then ask them to write a response to this prompt: When we multiply **whole numbers**, our product is greater than either of the numbers we multiply. But when we multiply fractions, all of our answers are less than 1. Why?

Extension activity

- Tell the students that there will be a "Fraction Party" at the conclusion of the unit on fractions and that another class (that is also studying fractions) will join you. Explain that the other class will bring drinks and your class will supply cookies. Then, distribute copies of cookie and frosting recipes. (Half of the class will bake cookies and the other half will frost them. Cookies will be decorated at school.) Have students determine how many cookies and how much frosting will be needed for the party, and whether their recipes will need to be reduced by $1/2$, $1/4$, or some other amount. Finally, have students calculate how much of each ingredient will be needed in the revised recipes.

Connections to other subjects

Language Arts. Have students write a newspaper article about the "Fraction Party" project. Submit it with photographs to the local newspaper.

Art. Decorate cookies or pictures using a fractions theme.

Home connections

Instruct students to explain how to multiply fractions to family members, then ask them to assist in baking the cookies and preparing the frosting for the party.

Resources for teachers

Fractions, Decimals, Ratios and Percents by Barnett, Goldstein, and Jackson (Heinemann, 1994)
Everyday Math—Grades 4–6 by Marge Lindskog (Frank Schaffer Publications, Inc., 1996)
Mega-Fun Math Games by Dr. Michael Schiro (Scholastic, 1995)
Math Activities by Robyn Silbey (Frank Schaffer Publications, Inc., 1996)

Multiplying Fractions Worksheet

Name ____

Solve the following problems. Write your answers in lowest terms.

1. $^1/_2$ x $^1/_2$ = ________
2. $^1/_2$ x $^1/_3$ = ________
3. $^4/_5$ x $^1/_6$ = ________
4. $^2/_3$ x $^3/_4$ = ________
5. $^9/_{10}$ x $^1/_8$ = ________
6. $^5/_9$ x $^1/_3$ = ________
7. $^2/_3$ x $^5/_8$ = ________
8. $^6/_7$ x $^5/_6$ = ________
9. $^{11}/_2$ x $^4/_5$ = ________
10. $^1/_2$ x $^5/_6$ = ________
11. $^{13}/_{15}$ x $^2/_4$ = ________
12. $^1/_3$ x $^6/_7$ = ________
13. $^7/_{10}$ x $^5/_6$ = ________
14. $^1/_4$ x $^5/_8$ = ________
15. $^3/_5$ x $^7/_8$ = ________
16. $^5/_6$ x $^5/_6$ = ________
17. $^1/_{10}$ x $^4/_5$ = ________
18. $^7/_8$ x $^5/_7$ = ________
19. $^1/_5$ x $^5/_7$ = ________
20. $^4/_9$ x $^3/_5$ = ________

Bonus: In the space below, draw a square and color it to illustrate $^1/_2$ x $^3/_4$.

Paper-folding Magic

About this learning activity

Students will fold and shade a sheet of paper to create equivalent fractions.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III P	V ○	VII ○
II ○	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

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Materials

- Multiplication Facts worksheet (for the engagement activity)
- $8^{1}/_{2}$ x 14-in paper (or any rectangular-shaped paper)
- Crayons

Engagement activity

As students arrive in the classroom, have them begin working on the Multiplication Facts worksheet. (Alternative: Assign the worksheet as homework the evening before the lesson.) Help students confirm that they completed the worksheet correctly by checking the question at the bottom of the worksheet. (Answer: What are equivalent fractions?) Explain that the next exercise will help them answer that question.

Exploration activity

1. Distribute a sheet of paper to each student.

2. Ask each student to fold the paper in half, then unfold it and shade 1 of the 2 sections with a crayon.

3. Ask students to name the fraction represented by the shaded area. (Answer: $^{1}/_{2}$.) Write $^{1}/_{2}$ on the board or a transparency.

4. Have each student refold the paper along the original crease, then fold it in half again in the same direction.

5. Instruct students to unfold their papers again. Ask them to name the number of shaded parts and the total number of parts. (Answers: 2, 4.)

6. Ask a student volunteer to write the fraction now represented (answer: $^{2}/_{4}$) next to the $^{1}/_{2}$ already on the board or transparency.

7. Have each student refold the paper along the 2 creases already made, then fold it in half again in the same direction.

8. Instruct students to unfold their papers and name the fraction represented now. (Answer: $^{4}/_{8}$.)

9. Ask another student volunteer to write this fraction next to the $^{2}/_{4}$ and $^{1}/_{2}$.

10. Facilitate a discussion with these and other questions:

- What happened when you folded the paper a second time? (Answer: Instead of 2 sections, we had 4. But the shaded area remained the same **size**.)

- What happened to the fraction representing the shaded part? (Answer: The numerator and denominator doubled.)

- What happened when the paper was folded a third time? (Answer: The shaded area remained the same size, but the number of sections doubled. And the numerator and denominator doubled again.)

12. Direct students' attention to the board or transparency where the fractions created in the exercise are written. Ask them to work in pairs to identify a pattern, then predict what would happen if the paper were folded another time.

13. Ask volunteers for their predictions. Fold the paper again to check the predictions, and write the new fraction ($^8/_{16}$) on the board or transparency next to $^4/_8$.

14. Ask the students to predict whether or not they could find equivalent fractions by multiplying the numerator and denominator of $^1/_2$ by 3 or another number. Then, test the theories given by demonstrating the multiplication on the board or transparency. Help students recognize the math principle that multiplying **both** the numerator and denominator **by the same number** always results in an equivalent fraction.

15. Distribute another sheet of paper to each student. Repeat this activity, having students fold their papers by thirds.

Evaluation

Have students write 3 equivalent fractions for each of these fractions: $^1/_4$, $^2/_3$, $^1/_5$, $^3/_4$, and $^5/_6$.

Journal assignment
Have students answer these questions in their journals:

- Why are $^4/_8$ and $^8/_{16}$ not found on a ruler?

- Why might this information be important to a house builder?

- To what other occupations might this knowledge be important?

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Extension activity

- Conduct a lesson about writing fractions in lowest terms (i.e., reducing, simplifying). Help students recognize that this activity also results in equivalent fractions—they are **dividing** the numerator and denominator **by the same number**.

Connections to other subjects

Art. Have students create designs that are made up of equivalent fractional parts.

Music. Discuss how equivalent fractions are used in music (e.g., 2 half notes equal 1 whole note).

Language Arts. Create reading, writing, drama, and other language arts assignments from the following list of books:

- *Fractions Are Parts of Things* by R. Dennis (Thomas Y. Crowell, 1971)
- *Frog Math Power, Level D* by Mary Jo Hand (Frog Publications, 1995)
- *Mad Math* by Sue Macy (Scholastic, 1987)
- *Math Activities* by Robyn Silbey (Frank Schaffer Publications) See pp. 30, 48–49.

Home connections

Instruct students to search newspaper and magazine articles for fractions, cut them out, and make a collage of equivalent fractions.

Resources for teachers

The Intermediate Mailbox (December/January 1993–94) by the Education Center, Inc., Greensboro, NC

Worksheet Magazine (November/December/January, 1988–89 and February/March 1987) by the Education Center, Inc., Greensboro, NC

Multiplication Facts

Name ____________________________

Instructions:

Solve each problem. Write the letter for each problem in the corresponding blank below. The first problem has been completed for you.

F	7 x 4 =	28	A	9 x 5 =	________	E	6 x 4 =	________
R	8 x 8 =	________	C	3 x 6 =	________	H	7 x 9 =	________
S	11 x 4 =	________	Q	9 x 4 =	________	E	6 x 4 =	________
U	9 x 9 =	________	T	6 x 7 =	________	W	4 x 4 =	________
I	3 x 9 =	________	O	7 x 8 =	________	N	4 x 8 =	________
L	5 x 5 =	________	H	7 x 9 =	________	V	5 x 8 =	________

_ _ _ _   _ _ _   _ _ _ _ _ _ _ _ _ _   F _ _ _ _ _ _ _ _?

16 63 45 42   45 64 24   24 36 81 27 40 45 25 24 32 42   28 64 45 18 42 27 56 32 44

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Comparing Fractions

About this learning activity

Students will compare fraction pairs to identify equivalent and larger/smaller fractions.

Process skills

○ Building models	○ Interpreting data
○ Categorizing or classifying	○ Measuring
○ Communicating	○ Observing
○ Comparing	○ Ordering
○ Controlling variables	○ Predicting
○ Experimenting	○ Reasoning
○ Hypothesizing	○ Recognizing relationships
○ Inferring	○ Recording

Ohio mathematics proficiency strand(s)

I ○	III P	V ○	VII ○
II ○	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Hint: You can create the fraction flash cards from the samples provided on pp. 126–129.

Materials

- Fraction Illustration sheets (see Explanation section)
- Fraction flash cards
- Comparing Fractions worksheet

Engagement activity

1. Help students discuss the concept of equivalent fractions by asking them how they might compare fractions to one another. Then distribute the fraction sheets and 2 flash cards to each student. Explain to the students that the fraction sheets can be used to represent any fraction.

2. Use a student's flash cards and your own set of fraction sheets to demonstrate this method of comparing fractions. For example, have students suppose that the cards drawn were 4/9 and 5/6. Explain that the ninths and sixths fraction sheets must be used. Have students place these fraction sheets side by side, then determine the size of 4/9 by counting down 4 on the ninths sheet. Explain that this distance represents 4/9.

3. Have students attempt to find the size of 5/6, providing direction, as needed. (Answer: The students should count down 5 on the sixths sheet.)

4. Next, have students compare the 2 fraction sheets. Help them recognize that 5/6 is greater than 4/9.

5. Discuss equivalent fractions with these and other questions:

   ? Are these 2 fractions equivalent? Why or why not?

   ? Are 1/2 and 2/4 equivalent? Explain.

   ? Could we make 5/6 and 4/9 equivalent? If yes, how?

Exploration activity

1. Have students lay their fraction sheets side by side and locate equivalent fractions (e.g., 1/2 is the same as 2/4 is the same as 4/8).

2. After students have found several equivalent fractions, distribute more flash cards. Have them compare the fractions on their cards, recording them from smallest to largest. Repeat this step, if desired.

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3. Distribute the Comparing Fractions worksheet and have students follow the instructions. Monitor their work, providing assistance to students when they struggle.

4. Discuss the exercise with these and other questions:

   ? How many equivalent fractions did you find? Note: Go around the room and let those with the largest number list them for the class.

   ? Why is it important to know when a fraction is larger than another?

Variations: You may wish to cut the fraction sheets into strips for comparison purposes. By using different colors of paper, the students can identify how many blue strips equal the size of a certain number of red strips, and find the equivalent fractions. Also, you can begin this activity using only halves, thirds, and fourths and have the students work in pairs.

Real-life applications

Explain uses for comparing fractions at the grocery store. For example, determine the better value—$2/3$ lb of vegetables or $3/4$ lb, each costing $1.00. Which gives you more vegetables?

Explanation

Fraction sheets are $8^{1}/_{2}$ x 11-in sheets of paper that have been divided into equal parts. You will need 11 different sheets, as follows:

- 2 equal parts (halves)
- 3 equal parts (thirds)
- 4 equal parts (fourths)
- 5 equal parts (fifths)
- 6 equal parts (sixths)
- 7 equal parts (sevenths)
- 8 equal parts (eighths)
- 9 equal parts (ninths)
- 10 equal parts (tenths)
- 11 equal parts (elevenths)
- 12 equal parts (twelfths)

Fraction sheets can be found on pp. 130–140.

Students will use the fraction sheets to find equivalent fractions. For example, by placing the fourths sheet next to the halves sheet, they can see (by size) that $2/4$ is the same as $1/2$. In addition, students will compare fractions, as explained in the engagement activity, step 2.

Evaluation

Give the students a list of fractions to place in numerical order, smallest to largest. Let them use the fraction sheets, if desired.

Journal assignment
Have students explain under what circumstances they might need to find equivalent fractions and under what circumstances they might need to compare fractions.

Hint: Students can draw circles on clear plastic sandwich bags and cut them out for this exercise. If using this activity with lower grades, teachers may wish to prepare the "pizzas" for students before beginning the exercise.

Extension activity

- Have students draw 12 "pizzas" (circles) on clear acetate. Instruct them to divide the first pizza in half, the second in thirds, the third in fourths, and so forth. Let students use protractors and straight edges, if desired. Then, have students use their pizzas to find equivalent fractions by placing 1 pizza over top of another. Have them use this technique to identify and record several fractions that are greater than and less than one another as well.

Home connections

Ask family members to serve pie for dessert, and talk about the fractional portions of the pie as it is served and the fractional portion that is left after everyone gets a piece.

Comparing Fractions

Name ____

Directions: Choose 2 fraction flash cards at a time and compare the fractions. If they are the same, list the fractions with an equal sign between them. If they are unequal, determine which is greater and use the greater than sign (>) between the 2 fractions.

Example: $4/5 = 8/10$ (four-fifths equals eight-tenths)
$4/5 > 3/7$ (four-fifths is greater than three-sevenths)

1. ____
2. ____
3. ____
4. ____
5. ____
6. ____
7. ____
8. ____
9. ____
10. ____
11. ____
12. ____
13. ____
14. ____
15. ____

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Fraction Flash Cards

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$
$\frac{1}{10}$	$\frac{1}{11}$	$\frac{1}{12}$	$\frac{2}{3}$
$\frac{2}{4}$	$\frac{2}{5}$	$\frac{2}{6}$	$\frac{2}{7}$
$\frac{2}{8}$	$\frac{2}{9}$	$\frac{2}{10}$	$\frac{2}{12}$

$\frac{2}{12}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{3}{6}$
$\frac{3}{7}$	$\frac{3}{8}$	$\frac{3}{9}$	$\frac{3}{10}$
$\frac{3}{11}$	$\frac{3}{12}$	$\frac{4}{5}$	$\frac{4}{6}$
$\frac{4}{7}$	$\frac{4}{8}$	$\frac{4}{9}$	$\frac{4}{10}$
$\frac{4}{11}$	$\frac{4}{12}$	$\frac{5}{6}$	$\frac{5}{7}$

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$\frac{5}{8}$	$\frac{5}{9}$	$\frac{5}{10}$	$\frac{5}{11}$
$\frac{5}{12}$	$\frac{6}{7}$	$\frac{6}{8}$	$\frac{6}{9}$
$\frac{6}{10}$	$\frac{6}{11}$	$\frac{6}{12}$	$\frac{7}{8}$
$\frac{7}{9}$	$\frac{7}{10}$	$\frac{7}{11}$	$\frac{7}{12}$
$\frac{8}{9}$	$\frac{8}{10}$	$\frac{8}{11}$	$\frac{8}{12}$

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$\frac{9}{10}$	$\frac{9}{11}$	$\frac{9}{12}$	$\frac{10}{11}$
$\frac{10}{12}$	$\frac{11}{12}$	$\frac{2}{2}$	$\frac{3}{3}$
$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$
$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$	$\frac{11}{11}$
$\frac{12}{12}$			

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Halves

Fourths

Sixths

Eighths

Tenths

Twelfths

Fun with Conversions

About this learning activity

Students will explore with base ten strips to recognize the connection between decimals and fractions.

Process skills

○ Building models	○ Interpreting data
○ Categorizing or classifying	○ Measuring
○ Communicating	○ Observing
○ Comparing	○ Ordering
○ Controlling variables	○ Predicting
○ Experimenting	○ Reasoning
○ Hypothesizing	○ Recognizing relationships
○ Inferring	○ Recording

Ohio mathematics proficiency strand(s)

I ○	III P	V ○	VII ○
II ○	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Sets of base ten strips (see sample, p. 146)
- Conversions worksheet
- A $1 bill
- 4 quarters

Engagement activity

1. Hold up a dollar bill and explain that you owe someone $0.50. Ask students how many parts (i.e., what fraction) you should cut from the dollar bill to pay the person back. Students will probably tell you that this isn't possible, so ask them how you can illustrate $0.50 by **folding** the dollar bill. (Answer: Fold the dollar bill in half.)

2. Next, hand a student the dollar bill and hold 3 quarters in your hand. Ask students how to fold the dollar bill to represent your 3 quarters. (Answer: Fold the bill into fourths, then open it until $1/4$ remains folded.)

3. Explain to the students that they have been converting between fractions and decimals, and that they will do more conversions in the next exercise.

Exploration activity

1. Allow students to *freely explore* with base ten strips until they get used to them.

2. Tell students that the large square is the unit of 1.0 and that placing 10 of the strips together forms a large square in which each strip is worth 0.10.

3. Demonstrate for students that placing 100 of the smaller squares together forms a larger square in which each square is worth 0.01.

4. Distribute the Conversions worksheet. Work the first 6 problems with the class to confirm understanding. Note: Make sure that students recognize the difference between problems #5 and #6.

5. Once students can make different decimals with base ten strips, ask them to name the fractions represented by the square, strips, and smaller squares. (Answers: $1/1$, $1/10$, $1/100$.)

6. Have students complete problems #7 through #12 on the worksheet. Walk around the room, monitoring students' work to make sure their answers are correct.

Note: Help students understand the difference between problems #11 and #12, if needed.

7. Instruct students to complete problems #13 through #18 by converting the decimal to a fraction or the fraction to a decimal. If students experience difficulty, help them to identify denominators that convert to fractions easily (e.g., 10, 100). Then, have them multiply the denominator and numerator to create a fraction that converts easily to a decimal.

Real-life applications

Bring in some department store advertisement circulars and help students calculate the discounts (e.g., an item is 1/10th off). Ask them to name similar ways we use conversions in daily life.

Evaluation

Give students a different set of conversion problems to work **without using the base ten strips**.

Journal assignment
Ask students to explain the difference between 0.05 and 0.50, using fractions and decimals.

Extension activity

- Help students convert more difficult fractions to decimals, such as 1/3, 1/6, and 1/8.

Connections to other subjects

Music. Have students name the fraction represented by a quarter note, then name the decimal it represents.

Economics. Have students name the fraction for a certain amount of money, such as $0.50.

Science. As students are measuring different amounts of chemicals for an experiment, have them convert the amounts to decimals and fractions.

Home connections

Have students locate and record examples of fractions and decimals found in stores, at home, and on signs, then estimate each example's conversion.

Conversions

Name ___________________________________

Illustrate decimals by using and drawing base ten strips:

1) 0.5

2) 0.06

3) 0.65

4) 0.32

5) 0.70

6) 0.07

Illustrate fractions by using and drawing base ten strips:

7) $^{4}/_{10}$

8) $^{56}/_{100}$

9) $^{1}/_{100}$

10) $1\ ^{3}/_{100}$

11) $^{9}/_{100}$

12) $^{9}/_{10}$

Illustrate decimals or fractions by using and drawing base ten strips. Then, write the equivalent decimal or fraction next to each drawing.

13) $^{4}/_{5}$

14) 0.2

15) $^{8}/_{50}$

16) 0.75

17) $^{6}/_{25}$

18) 0.08

Conversions Answers

Illustrate decimals by using and drawing base ten strips:

1) 0.5

5 strips

2) 0.06

6 smaller squares

3) 0.65

65 smaller squares or 6 strips and 5 smaller squares

4) 0.32

32 smaller squares or 3 strips and 2 smaller squares

5) 0.70

7 strips or 70 smaller squares

6) 0.07

7 smaller squares

Illustrate fractions by using and drawing base ten strips:

7) $^4/_{10}$

4 strips or 40 smaller squares

8) $^{56}/_{100}$

5 strips and 6 smaller squares or 56 smaller squares

9) $^1/_{100}$

1 smaller square

10) 1 $^3/_{100}$

1 large square and 3 smaller squares

11) $^9/_{100}$

9 small squares

12) $^9/_{10}$

9 strips or 90 small squares

Illustrate decimals or fractions by using and drawing base ten strips. Then, write the equivalent decimal or fraction next to each drawing.

13) $^4/_5$

Converts to 0.8
8 strips or 80 small squares

14) 0.2

Converts to $^2/_{10}$
2 strips or 20 small squares

15) $^8/_{50}$

Converts to 0.16
1 strip and 6 small squares or 16 small squares

16) 0.75

Converts to $^3/_4$
7 strips and 5 small squares or 75 small squares

17) $^6/_{25}$

Converts to 0.24
2 strips and 4 small squares or 24 small squares

18) 0.08

Converts to $^8/_{100}$
8 small squares

Base Ten Strips

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Scoring High with Fractions and Decimals

About this learning activity

Students will use scores from the newspaper and their own basketball shots to calculate fractions and decimals.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III P	V ○	VII ○
II ○	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

Hint: You can substitute nerf balls and containers for basketballs and a basketball court.

- Basketballs
- Basketball court
- Calculators
- Paper and pencils
- Newspaper sports sections

Engagement activity

Take the students to a basketball court (indoors or outdoors). Have them work in pairs, taking turns shooting 10 times each and recording the baskets scored. (Or set up containers in the classroom and use soft balls to shoot 10 times.)

Exploration activity

1. Distribute newspapers. Have students work in groups to find and record sports statistics like basketball shooting percentages and baseball averages.

2. Have the groups discuss how their statistics might be calculated. Remind them to discuss their own statistics, recorded during the engagement activity, too.

3. Instruct groups to appoint a spokesperson. Have each spokesperson report the findings. Record pertinent facts on a transparency or the board.

4. Ask students to record their basketball results as a fraction (e.g., $^{7}/_{10}$, $^{5}/_{10}$).

5. Briefly review students' knowledge of decimals. Ask them to convert their basketball results to decimals (e.g., 0.7, 0.5).

6. Pose this question: What if you had 12 shots?

7. Explain that they could still write their results in fractions (e.g., $^{7}/_{12}$, $^{5}/_{12}$), then ask how they would change the fractions to decimals. Challenge groups to use calculators to find a mathematical operation for converting each member's original basketball score—out of 12 shots—to a decimal.

8. After a few minutes, have groups report what they have tried and record those attempts. Note: Through the process of elimination, someone will eventually find the correct mathematical operation—divide the numerator by the denominator.

9. Help students discuss how the group(s) found the correct mathematical operation. Then, let groups practice this operation on their members' scores.

Evaluation

Direct each student to make a list of 20 fractions, then trade lists with a partner. Have each student convert the 20 fractions to decimals.

Journal assignments

- Have students write about the relationship between fractions and decimals, and describe the method they used to change fractions to decimals.

- Instruct students to answer this problem in their journals: You run out of gas while mowing lawns. Your gas can holds 12 gallons. The gas pump displays quantities in decimals. What decimal quantity will the gas pump display after you fill your gas can?

Extension activity

☼ Instruct groups of students to create simple games in which each player has 7 opportunities to score a point. Then, let students play the games and record results, both as fractions and decimals.

Connections to other subjects

Language Arts. Choose a paragraph in a student text. Have students count the number of declarative, interrogative, imperative, and exclamatory sentences in the paragraph. Then, express each type of sentence as a fraction and a decimal (e.g., declarative: $^2/_{10}$, 0.2).

Home connections

Have each student list all the members of his/her household, including pets, then write a fraction and a decimal for the portion of the household that is female and the portion that is male.

Resources for teachers

Big Book of Everything—Fifth Grade by Mel Fuller (Instructional Fair, 1995)
Problem Solving in Sports (Frank Schaffer Publications, Inc., 1995)
Worksheet Magazine (April/May/June, 1992) by The Education Center, Inc., Greensboro, NC
Mad Math by Sue Macy (Scholastic, Inc., 1987)

What's Your Order?

About this learning activity

Students will use number lines and base ten strips to help them order fractions and decimals sequentially.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I X	III P	V ○	VII ○
II ○	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Base ten strips (see sample on p. 146)
- 2 number lines (for each student; see samples on pp. 156–157)
- Ordering worksheet
- 2 counters (e.g., bingo chips, bottle caps) (for each student)
- *War* cards (see *War* instructions, p. 152)

Engagement activity

1. Tell students to imagine themselves in the following scenario:

   When you go home today, you find leftover pizza in the refrigerator. Both you and your brother get to the refrigerator at the same time. Your brother asks if you prefer 1/4 or 0.3 of the pizza.

2. Ask students which they would choose. (Answer: 0.3 because it is larger than 1/4.)

3. Ask students to explain how they decided which portion of the pizza was larger. After some students explain the correct method, help the class to recognize that this method helped them to **order** 1/4 and 0.3, ordering 0.3 as larger than 1/4. Further explain that they will apply their ordering skills in the next exercise.

Hint: In the exploration activity, students will be comparing fractions to whole numbers and decimals. Remind them of their experiences with converting decimals to fractions. Encourage them to say the decimals aloud so they can hear the tenths and recognize the fraction (e.g., 0.2 is two-tenths or 2/10). Suggest that students write the fractions below each decimal interval on the number line (e.g., 1.2 is 1 2/10, 1.4 is 1 4/10, 1.6 is 1 6/10, 1.8 is 1 8/10).

Exploration activity

1. Refresh students' memories concerning the meanings of the greater than, less than, equal to, and related symbols (i.e., $<$, $>$, $=$, $\leq$, or $\geq$). Distribute the 2 number lines, 2 counters, and an Ordering worksheet to each student.

2. Instruct the students to complete problems #1 through #5, using number line 1. Demonstrate the process, if necessary, by placing a counter on each number in the problem, then observing the line to make a comparison. Example: For problem #1, put counters on the number 2 and the number 1. Explain that by observing the number line, they can see that 2 is greater than 1; therefore, the $>$ sign is used to compare the 2 numbers.

3. Instruct students to complete problems #6 through #10, still using number line 1 and the counters.

Hint: If students haven't used base ten strips before, give them time to freely explore with the strips first. Be sure they recognize that the large strip represents 1 (each small square represents .1, with 10 in a strip totaling 1) and that the smallest squares represent .01, with 100 of them totaling 1.

4. Distribute the base ten strips. Instruct students to complete problems #11 through #24, using both the base ten strips and number line 2. Explain that to solve the problems, students need to probe them with their base ten strips and their number lines.

5. If students experience difficulty with problems #16 through #24, remind them that they can display the fractions with their base ten strips.

6. As a culminating activity, distribute index cards (or pieces of paper) and let the students play *War* with fractions, decimals, and whole numbers. Instruct them to use number lines, scratch paper, and base ten strips, as needed, throughout the game.

War *Instructions:* *Create the deck of cards with these numbers: 3/4, 1/5, 1/10, 6/10, 3/5, 8/10, 1/2, 5/10, 1/100, 50/100, 45/100, 2/10, 9/10, 1 8/10, 1.0, 2.0, 2.3, 1.6, 0.2, 0.7, 0.9, 2.4, 0.01, 0.21, 0.45, 0.09, 1.78, and 0.04. Have students play by the conventional* War *rules, i.e., play with a partner, each turning over a single card at a time, and the person with the highest card keeps both cards. If equivalent cards turn up, players lay 2 more cards face down, then lay a third card face up and the highest third card wins. (If the third cards are equivalent too, repeat until the player with the highest card takes all.) The winner is the first player to collect all of the cards, or the player with the most cards when time is called.*

Real-life applications

Encourage students to work with decimals to the hundredths place, using money examples they might find in daily life.

Evaluation

- Review students' answers on the Ordering worksheets (see worksheet answers on p. 155), their base ten strip comparisons, and their verbal expressions of numbers in both decimal and fraction forms.

- Have students draw base ten strips to prove their number comparisons.

Journal assignment
Ask students to explain why it is important to put numbers in sequential order. Then, have them explain the difference between 0.5, 0.50, and 0.500.

Extension activities

- ☼ Have students order some or all of the War cards in the deck.

- ☼ Continue plotting decimals on number lines, using thousandths, ten-thousandths, and hundred-thousandths places.

Connections to other subjects

Language Arts. Work with your librarian to arrange for students to order books by catalog numbers for reshelving.

Home connections

Have students look for and record examples of ordering in their homes (e.g., measuring cups, wrench sets).

Ordering Worksheet

Name ______________________________

Instructions

Fill in the blanks with the appropriate symbol: <, >, =, ≤, or ≥.

1) 2 ____ 1

2) 1 ____ 1.4

3) 0.6 ____ 0.8

4) 0.2 ____ 1

5) 1.8 ____ 1.2

6) $1\frac{4}{10}$ ____ 2

7) $\frac{6}{10}$ ____ $1\frac{6}{10}$

8) $\frac{4}{10}$ ____ $\frac{8}{10}$

9) $\frac{5}{10}$ ____ $\frac{3}{10}$

10) $1\frac{1}{10}$ ____ $\frac{9}{10}$

11) 0.50 ____ 0.25

12) 0.30 ____ 0.75

13) 0.55 ____ 0.50

14) 0.86 ____ 0.82

15) 0.3 ____ 0.35

16) $\frac{1}{2}$ ____ 0.50

17) $\frac{1}{4}$ ____ 0.27

18) $\frac{1}{5}$ ____ 0.13

19) 0.76 ____ $\frac{3}{5}$

20) $1\frac{2}{5}$ ____ 1.4

21) 0.5, 0.35, 0.76 ____ 0.35

22) 0.89 ____ 0.25, 0.6, 0.75

23) 0.25, $\frac{1}{4}$ ____ $\frac{2}{8}$

24) 0.21, 0.3, 0.45 ____ 0.45

Ordering Worksheet Answers

1) $2 > 1$

2) $1 < 1.4$

3) $0.6 < 0.8$

4) $0.2 < 1$

5) $1.8 > 1.2$

6) $1\,^4/_{10} < 2$

7) $^6/_{10} < 1\,^6/_{10}$

8) $^4/_{10} < {}^8/_{10}$

9) $^5/_{10} > {}^3/_{10}$

10) $1\,^1/_{10} > {}^9/_{10}$

11) $0.50 > 0.25$

12) $0.30 < 0.75$

13) $0.55 > 0.50$

14) $0.86 > 0.82$

15) $0.3 < 0.35$

16) $^1/_2 = 0.50$

17) $^1/_4 < 0.27$

18) $^1/_5 > 0.13$

19) $0.76 > {}^3/_5$

20) $1\,^2/_5 > 1.4$

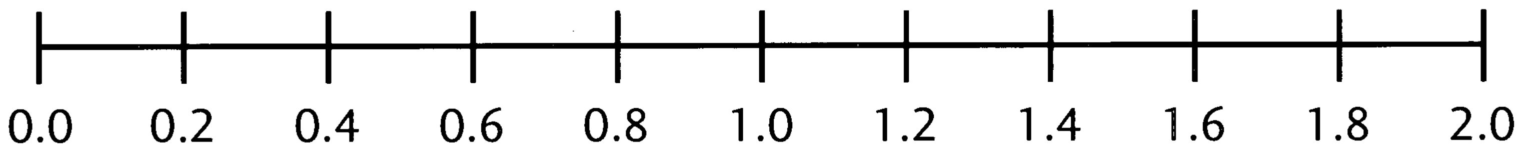
21) $0.5, 0.35, 0.76 \geq 0.35$

22) $0.89 > 0.25, 0.6, 0.75$

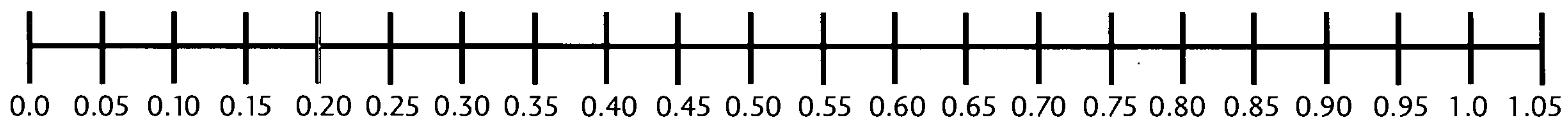
23) $0.25, {}^1/_4 = {}^2/_8$

24) $0.21, 0.3, 0.45 < 0.45$

Number Line 1



Number Line 2



Equality—More or Less

About this learning activity

Students will create whole number, fraction, and decimal cards, then compare them to determine if they are equal to, greater than, or less than each other.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III P	V ○	VII ○
II ○	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- 6 index cards (for each student)
- Paper and pencils (for each student)
- 2 dice

Engagement activity

Distribute 6 index cards to each student. Instruct them to write fractions on 2 cards, decimals on 2 cards, and whole numbers on 2 cards.

Exploration activity

1. Instruct the students to trade cards with a partner and compare the fractions, decimals, and whole numbers on the cards they receive. Have them record the comparisons, using the symbols $<$, $>$, $=$, $\leq$, and $\geq$.

2. Have students repeat the exercise several times by trading cards with new partners. Note: Give students plenty of time to work on this step and step 3.

3. Ask each student to arrange all 6 of the cards from the final trade from largest to smallest, then record the 6 numbers in **sequential order**.

4. Instruct student pairs to write their quantities on the board and explain the process they used to order them.

5. After several partners have presented their numbers and explanations, use the last set presented for a dice game. Number the set 1 through 6. Roll a die twice, writing the corresponding fraction, decimal, or whole number on the board. Compare the 2 quantities and have students select the appropriate symbol to put between them.

6. Let students take turns rolling the die and leading the class in comparing the amounts and solving the problem. Use different card sets to give more students the opportunity to facilitate the game.

Evaluation

Have 2 to 3 students combine their cards and spread them out face down on the table. Instruct each student to choose 2 cards and record their numbers on paper, using the comparison symbols. Have them continue until all cards have been used.

Journal assignments

- Instruct students to record the meanings of the symbols <, >, and = in their journals.
- Have students respond to the questions in this scenario:

  At the Yum-Yum Bakery you work a variety of shifts—day shift (8 hours @ $6.25 per hour), evening shift (6 hours @ $6.75 per hour), or night shift (5 hours @ $7.25 per hour).

  ? How much do you earn on each shift?

  ? Write comparisons of the shift salaries, using the appropriate symbols.

  ? Why do you think night workers make more per hour?

Extension activities

☼ Collect all of the students' index cards. Distribute a card to each student. Choose 1 of the remaining cards and write its number, fraction, or decimal on the board. Ask all students who are holding a quantity that is **greater than or equal to** this number to stand. Check their cards to ensure accuracy. Repeat the task for students whose cards are **less than or equal to** your number. Repeat this activity with other cards in the leftover pile as time allows.

☼ Collect all the students' cards. Let students play *War* with this deck during free times.

Connections to other subjects

Music. Ask the music teacher to discuss how note values change based on the time signature. Have the students compare note values, using the appropriate symbols.

Resources for teachers

Everyday Math—Grades 4–6 by Marge Lindskog (Frank Schaffer Publications, Inc., 1996)
Mega-Fun Math Games by Michael Schiro (Scholastic, Inc., 1995)
Math Activities by Robyn Silbey (Frank Schaffer Publications, Inc., 1996)

Sweet Ratios

About this learning activity

Students will use quantities of multi-colored candies to create and study ratios.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III P	V ○	VII ○
II ○	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Hint: You may wish to have students bring candy from home for this exercise.

Materials

- Colored candies (e.g., M&Ms, Skittles)
- Inventory worksheet
- Paper plates (to hold the candies)

Engagement activity

Distribute Inventory worksheets, paper plates, and candies. As you review the worksheet with students, discuss the concept of *inventory* and provide a number for instruction #5 on the worksheet (i.e., number of parts needed today).

Exploration activity

1. Instruct students to follow the instructions on the Inventory worksheet for completing the **first 2 columns and item #9 only**.

2. Call time and ask students to draw a line below their last observations.

3. Using the board or overhead projector, compile students' observations. Add 1 observation per student. Have students record the compilation on their worksheets. Instruct them to put a check beside the observations if they have already listed them, and add any observations to the bottom of their lists (beneath the lines they drew) if they didn't make them while completing the exercise.

4. Define **ratio** for the students: comparing 2 quantities. Illustrate ways to write a ratio, using these and other examples:

    - Words like *to* and *out of* (red to blue, 4 to 3, 4 out of 20)
    - Fraction ($^4/_3$)
    - Colon (4:3)
    - Percent (20%)
    - Decimal (0.2)

5. Using the third column of the Inventory worksheet, write a **part-to-whole** ratio that relates to the candies students counted and recorded. Write the ratio with the word *to*. (Example: 5 red candies to 25 total candies). Have students complete the third columns on their worksheets.

6. In the fourth column, rewrite the ratio you used in step 5 in the colon form. Instruct students to complete column 4 on their worksheets.

7. In the fifth column, write the ratio in fraction form. Note: Do not reduce fractions. Tell students to complete the fifth column on their worksheets, without reducing fractions.

Evaluation

Circulate and note each student's understanding of ratios as the class completes candy trades.

Candy trading instructions:

1. *Explain that for this exercise, **equal ratios** can be traded.*
2. *Distribute 1 index card per candy color to each student. Have them print (in letters large enough to be seen across the room) each candy color at the top of each card, then write the fraction form of the ratio (equally large) on each card.*
3. *Choose a student at random, and have him/her initiate a trade. Explain that the trader decides which color he/she wants from another student, but the desired color must be different from the color the trader wishes to trade. (Example: Red can be traded for blue.)*
4. *Instruct the trader to hold up the card that displays the ratio for this trade.*
5. *Explain that any students who are in a position to complete the trade should stand and hold up the card(s) that equals the ratio on the trader's card.*
6. *Have the trader choose the card that suits him/her, and complete the trade by exchanging candy and cards.*
7. *Choose a new trader and continue the exercise.*
8. *Explain that traders must adjust the ratios displayed on their other cards after completing a trade.*

Trade exercise options:

- *Each student may participate in 1 trade only.*
- *Trades proceed until everyone has the same number of colors.*
- *Trades proceed until everyone is left with a predetermined number of colors.*
- *Materials can be put away for completion at a later time.*
- *Set a time limit, and the person with the most of 1 color wins.*

Journal assignment
Have students explain why trading for something you want can be better than purchasing it.

Connections to other subjects

Social Studies. Discuss bartering and trading as an alternative to monetary acquisition of goods.

Home connections

Ask students to conduct a taste test for peanut butter and jelly sandwiches at home.

Taste test instructions:

1. *Assign 1 ratio to each student from this list: 1 to 1, 2 to 1, 1 to 2, 2 to 3, 3 to 2, 3 to 1, 1 to 3, 4 to 1, and 1 to 4.*
2. *Explain that the ratio represents the number of* tablespoons of peanut butter *to the number of* tablespoons of jelly.
3. *Instruct students to make at least 1 sandwich with their assigned ratios, cut their sandwiches into 9 pieces (samples), and put the samples in a sealed sandwich bag with the ratio written on the outside of the bag.*
4. *Have each student bring his/her sandwich and a small bottle of water to class the next day.*
5. *Set up a taste-testing station in the classroom. Have students leave their sandwiches and bottles of water at the station as they arrive.*
6. *Distribute a Taste Test Checklist (p. 166). Have each student taste and rate several sandwiches, and record the results for each. Remind students to record the ratio (i.e., written on the outside of the plastic bags) for each taste test on their checklists.*
7. *Compile the results as a class.*

Inventory Worksheet

1. Person doing the inventory: ______________________________

2. Date of inventory: __________________

3. Name of item inventoried: ______________________________

4. Total number of "parts" (pieces of candy): ________

5. For today's exercise, your number of parts needs to be: _____

   Note: After making sure you have the amount of inventory required, you can eliminate the extra inventory by quietly eating it.

6. List the different color names in the first column below.

7. Place the number of each color in the second column.

8. Skip to item 9. Await further instructions before completing columns 3 through 5.

Color	Number			
____________	________	______ to ______	______ : ______	________
____________	________	______ to ______	______ : ______	________
____________	________	______ to ______	______ : ______	________
____________	________	______ to ______	______ : ______	________
____________	________	______ to ______	______ : ______	________
____________	________	______ to ______	______ : ______	________

9. On the back of this worksheet, make as many math observations as possible about what you have at your desk.

Taste Test Checklist

Ratings

1 - Nasty 2 - Yuck, But Edible 3 - Not Bad 4 - Good, This'll Do 5 - Yum

Ratio

1:1	1	2	3	4	5
2:1	1	2	3	4	5
1:2	1	2	3	4	5
2:3	1	2	3	4	5
3:2	1	2	3	4	5
3:1	1	2	3	4	5
1:3	1	2	3	4	5
4:1	1	2	3	4	5
1:4	1	2	3	4	5

Currency Conversions

About this learning activity

Students will compare and convert between U.S. dollars and foreign currencies.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III P	V ○	VII ○
II ○	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Table of Foreign Exchange Rates
- Foreign Exchange Conversions worksheet
- Exchanging U.S. Dollars worksheet
- What's It Cost? worksheet
- Daily newspapers
- Calculators

Engagement activity

Have students bring in daily newspapers for several days, including *The Wall Street Journal* and other national papers. Explain the concepts behind foreign currency and exchanges (see Explanation section). Then, distribute copies of financial pages (from the newspapers you collected) that contain exchange rate tables and let students freely explore them. For the next several days, keep a class chart of daily exchange rates and help students observe the fluctuations.

Exploration activity

1. Distribute a copy of the Table of Foreign Exchange Rates to each student. Help students understand how to read it with the following explanation:

   The table shows the name of the country, the number of U.S. dollars needed to buy 1 unit of that country's currency, and the number of units of the foreign country's currency needed to buy 1 dollar. Some newspapers list the name of the basic currency unit for the country too. While the names of the currency unit may be the same in several countries, their values are different.

2. Then, demonstrate for students that monetary exchange rates from 1 country to another can be calculated with the use of a proportion problem.

   Example: $\dfrac{1 \text{ U.S. dollar}}{1.92 \text{ British pound}} = .519 \text{ U.S. dollars per British pound}$

3. Organize students into small groups and have them practice proportion problems by completing the worksheets provided with this lesson.

4. Facilitate a class discussion with these and other questions:

   ? If you visit Niagara Falls and decide to spend the night, is it less expensive to stay on the U.S. side or the Canadian side? Note: Be sure students understand that Niagara Falls is accessible from both countries.

? How could you determine the best buy for food and lodging?

? How would this skill help you to better plan a vacation to another country? To save money on a trip?

Real-life applications

Let students prepare for a trip to a foreign country. Help them work out the details, e.g., choose a country; gather information about language, currency, exchange rates, and places of exchange (in the U.S. and abroad); choose what to take with them and what to purchase there; make travel arrangements to the country and inside it; study costs in the host country to determine how much money to take.

Explanation

In early history, livestock and other items of intrinsic value were exchanged directly. As trade became more complex, trade became an inconvenient means of acquiring goods. Therefore, governments began creating tokens of monetary value to be used as a medium of exchange.

Today, most countries have their own monetary systems. Governments produce coins and/or paper currency, which is used as the basis for exchange. A currency's value differs from country to country, can fluctuate dramatically, and is defined by the functioning of the foreign exchange market. This market is not a place or organization. Rather, it is a tracking of exchanges made by financial institutions like large commercial banks. The exchange rate between 2 currencies, therefore, fluctuates constantly, much like the price of stocks on the stock market.

A person traveling to another country needs to understand the currency system of that country and monitor the exchange rates between his or her home country and the destination. Current exchange rates are listed daily in many major newspapers.

Evaluation

- Observe students as they work in their small groups to complete the worksheets.

- Have students choose a country and list several varied activities available in the country. Then, instruct them to **approximate** the cost in U.S. dollars and **compute** the cost in the currency of the chosen country.

Journal assignment

Ask students to explain what factors might cause a country's currency rates to fluctuate.

Extension activities

- Have the students choose 2 different countries and compare the rates of exchange of these countries to one another.
- Help students visit the Universal Currency Converter™ found on the web at http://www.xe.net/currency/classic/.

Connections to other subjects

Social Studies and Math. Have the students choose a country and track the exchange rate over a specific period of time. Then, instruct them to graph the information and compare their results. In addition, students could determine the average rate of exchange for their particular country.

Social Studies. Have students research the figure on a specific piece of currency.

Art. Tell students to design a piece of currency for a new country and give an explanation of the design (e.g., What is it? Why did they choose this design?).

Home connections

Ask families to let students plan a meal that features food from another country, then help students shop for the necessary food. Have students track the cost of the meal and calculate its cost in the currency of the featured country.

Resources for teachers

Xenon Laboratories Currency Update Service

Note: Subscribe at http://www.xe.net/currency/subscrib.htm. The master site is located at http://www.xe.net/currency/. The subscription (a mailing) provides a listing of currency exchange rates each business day (Monday through Friday), using the noon rate of the previous business day. For more information about the service, including rate accuracy and notes on significant figures, go to http://www.xe.net/currency/about.htm.

Table of Foreign Exchange Rates

Country	Current Unit	Foreign Currency in U.S. Dollars	U.S. Dollar in Foreign Currency
Australia	dollar	0.722	1.385
Austria	schilling	0.097	10.26
Belgium	franc	0.033	30.14
Britain	pound	1.928	.519
Canada	dollar	0.837	1.19
Denmark	krone	0.177	5.65
Egypt	pound	0.305	3.25
France	franc	0.202	4.956
Germany	mark	0.684	1.462
Israel	shekel	0.407	2.455
Italy	lira	0.0009	110.0
Japan	yen	0.0079	125.60
Mexico	peso	0.0003	3226.00
Spain	peseta	0.0106	94.00
Switzerland	franc	0.761	1.313

Note: Some newspapers show the name of the basic currency unit for the country. While the names of the currency units may be the same in several countries, the values are different. The exchange rate between 2 currencies fluctuates constantly, much like the price of stocks on the stock market. But this table can be used as a guide for activities in this lesson.

Foreign Exchange Conversions

Name ______________________________

Instructions

1. Use the Table of Foreign Exchange Rates to determine an approximate ratio between a U.S. dollar and 1 unit of the specified foreign currency.
2. Use this ratio to **estimate** the cost of the purchase shown, without using pencil or calculator.
3. Calculate the cost in U.S. dollars.
4. Compare your estimate to the actual cost.

Purchase Amount	Ratio (approximate)	Estimate	Actual Value
30 British pounds			
200 French francs			
45 German marks			
300 Belgian francs			
500 Spanish pesetas			
70 Austrian shillings			
80 Swiss francs			
60 Canadian dollars			
40 Saudi Arabian riyal			
7000 Italian lira			

Foreign Exchange Conversions
Answers

Purchase Amount	Ratio of U.S. to Foreign Values (approximate)*	Estimate*	Actual Value
30 British pounds	.7 (or 3/4):1		30 x .722 or $21.66
200 French francs	.2:1		200 x .202 or $40.40
45 German marks	.68 (or 2/3):1		45 x .684 or $30.78
300 Belgian francs	.33 (or 1/3):1		300 x .033 or $9.90
500 Spanish pesetas	.01:1		555 x .0106 or $5.30
70 Austrian shillings	.1:1		70 x .097 or $6.79
80 Swiss francs	.75 (or 3/4):1		80 x .761 or $60.88
60 Canadian dollars	.8:1		60 x .837 or $50.22
40 Saudi Arabian riyal	.25 (or 1/4):1		40 x .267 or $10.68
7000 Italian lira	.001:1		7000 x .0009 or $6.30

*Student answers will vary.

Exchanging U.S. Dollars

Name ______________________________

Instructions

For each of the listed currencies, calculate the number of units that you would receive when exchanging 100 U.S. dollars.

Currency	Value When Exchanging 100 U.S. Dollars
Swiss franc	
Belgian franc	
French franc	
German mark	
Mexican peso	
Spanish peseta	
Italian lira	
Danish krone	
British pound	
Egyptian pound	
Austrian schilling	
Israeli shekel	

Exchanging U.S. Dollars
Answers

Currency	Value When Exchanging 100 U.S. Dollars
Swiss franc	100 x 1.313 or 131.3 francs
Belgian franc	100 x 30.14 or 3014 francs
French franc	100 x 4,962 or 496.2 francs
German mark	100 x 1.462 or 146.2 marks
Mexican peso	100 x 3226 or 322600 pesos
Spanish peseta	100 x 94 or 9400 pesetas
Italian lira	100 x 110 or 11000 lira
Danish krone	100 x 5.65 or 565 krone
British pound	100 x .519 or 51.9 pounds
Egyptian pound	100 x 3.25 or 325 pounds
Austrian schilling	100 x 10.26 or 1026 schillings
Israeli shekel	100 x 2.455 or 245.5 shekels

What's It Cost?

Name ___________________________________

Instructions

In the second column, write the name of the country referred to in the question (first column). Then, use the Exchange Rate Table to calculate the cost of each purchase in U.S. dollars. Show all of your work.

Purchase	Name of Country	Cost in U.S. Dollars
Lunch at a London pub costs 3 pounds		
A hotel room in Paris costs 700 francs per night		
Tickets to a theatre in Toronto cost 47 dollars		
A bottle of tequila in Mexico City costs 15,000 pesos		
A camel ride in Cairo costs 4 pounds		
A ticket to a concert in Vienna costs 125 schillings		
A Toyota in Tokyo costs 8000,000 yen		
A ticket to a bullfight in Madrid costs 1000 pesetas		
A box of chocolates in Geneva costs 11 francs		
An all-day tour of Rome costs 30,000 lira		

What's It Cost?
Answers

Purchase	Name of Country	Cost in U.S. Dollars
Lunch at a London pub costs 3 pounds	Britain	$5.78
A hotel room in Paris costs 700 francs per night	France	$141.40
Tickets to a theatre in Toronto cost 47 dollars	Canada	$39.34
A bottle of tequila in Mexico City costs 15,000 pesos	Mexico	$4.50
A camel ride in Cairo costs 4 pounds	Egypt	$1.22
A ticket to a concert in Vienna costs 125 schillings	Austria	$12.13
A Toyota in Tokyo costs 8000,000 yen	Japan	$6320.00
A ticket to a bullfight in Madrid costs 1000 pesetas	Spain	$10.60
A box of chocolates in Geneva costs 11 francs	Switzerland	$8.37
An all-day tour of Rome costs 30,000 lira	Italy	$27.00

Notes

Strand IV: Geometry and Spatial Sense

Learning Activities **Page**

Notes

Which Way Did They Go?

About this learning activity

Students will create lines and identify them as parallel, perpendicular, intersecting, and so forth.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II ○	IV P	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials (for each student)

- Unlined paper
- Colored pencils (or fine-tip markers)
- Straight edge
- String (e.g., extra-long shoestrings)

Engagement activity

1. Distribute the unlined paper. Have students fold it in any manner they choose and as many times as they want (or as many times as you decide will best suit the needs of this activity—5 or 6 folds are usually sufficient). Tell students to firmly crease each fold.

2. Instruct students to open their papers and flatten them with their hands, then use straight edges and colored markers to trace their creases.

Hint: If students name terms that they've heard but are unfamiliar with, have them look up the words and, as a class, develop working definitions.

3. Ask for observations as students work. (Answers could include types of lines, angles, or shapes being formed.) List the observations down the side of the board as students name them.

4. As students identify the different kinds of lines represented on their papers, you may wish to have them repeat the activity, using different colors for the different kinds of lines (e.g., parallel lines in red, intersecting lines in green).

5. If desired, discuss different kinds of angles (e.g., right, acute, obtuse, complimentary, supplementary) and different shapes (e.g., triangles, quadrilaterals) with students too.

Exploration activity

1. Arrange students in pairs. Give each student a string.

2. Ask pairs to volunteer. One pair at a time, have them use their strings to demonstrate a type of line—intersecting, perpendicular, or parallel.

3. After each demonstration, have the class (in their pairs) practice the line demonstrated. Circulate through the room to check their work.

4. Ask for observations and record them on the other side of the board (beside the list developed in the engagement activity). *(Hint: If students name terms that they've heard but are unfamiliar with, have them look up the words and, as a class, develop working definitions.)*

5. If desired, repeat this activity using types of angles and shapes.

6. Using the lists on the board, discuss the concept of plane surface versus space. Correct the lists and make additions, if needed.

7. Help students conclude that lines do not have any dimension other than length, and dimension is unlimited.

Evaluation

While you hold 1 of 2 strings for reference, have students demonstrate their understanding of intersecting, perpendicular, and parallel lines by holding their strings appropriately.

Journal assignment
Ask students to explain the phrase, "Infinity can only be conceived of in the mind," in their journals.

Extension activities

- String fish line across an open space. Use spring clothespins to hang sheets of 8 1/2 x 11-in and 8 1/2 x 14-in paper. Then, discuss lines and planes with the class by relating the concept of plane to volleyball (i.e., plane of the net) and football (i.e., plane of the goal line).

- View *Donald in Math Magic Land* (Walt Disney Home Video, 27 minutes).

Connections to other subjects

Art. Using balloons, string, and glue, have students create space by following these instructions:

1. Blow up a balloon.
2. Dip lengths of string into watered-down glue.
3. Drape strings over the balloon's surface.
4. Allow to dry overnight.
5. Burst the balloon.

Physical Education. Have students identify parallel, intersecting, and perpendicular lines that appear on various playing surfaces.

Curves Ahead!

About this learning activity

Students will plot points on a graph to create different shapes.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II ○	IV P	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Paper and pencils
- Ruled straight edge
- Unlined paper
- 20 x 20 Paper (See sample on p. 187.)
- 20 x 20 transparency
- Colored pencils
- Plotting Sets worksheet
- Overhead projector
- Graph paper

Engagement activity

Have students use ruled straight edges to create line segments of various lengths (e.g., 1 inch, 3 inches, $4\frac{1}{2}$ inches, $5\frac{1}{4}$ inches, $\frac{3}{4}$ inch, $\frac{1}{8}$ inch, longest possible line for the sheet of paper). Let them practice creating line segments until they can make them straight and neat, and can begin and end lines accurately (i.e., on the desired points).

Exploration activity

1. Distribute 20 x 20 Paper and the Plotting Sets worksheets. Demonstrate how to connect number sets with line segments by plotting problem #1 from the worksheet in the upper left corner of the 20 x 20 transparency.

2. As you finish plotting, ask students to identify what they observe.

3. Instruct students to complete problem #1 **in red** on their papers and record their observations about the figure created.

4. Have students follow the instructions on the worksheet to complete problems #2 through #4.

5. Instruct students to cut the 20 x 20 sheet into 4 equal sections, using the 10s as points of reference.

6. Have them arrange the 4 figures (i.e., 1–4) in a row from left to right, then compare them and record their observations.

7. Compile a list of student observations on the board. Have students put a check beside any items on their own lists that match those in the class list. Have them add to their lists any observations that they did not recognize. Instruct them to keep their lists for reference.

8. Tell students to write their names lightly on the back of each paper. Then, organize students into pairs and have them use their 8 figures to create various designs. Note: You may wish to recommend that students trim each figure to the number lines for this step.

9. Have each student choose his or her favorite design, transfer it to a sheet of graph paper, number it "5," and sign it on the back.

Evaluation

Collect figures 1–5, and give credit for work completed and turned in.

Journal assignment

Have students complete this sentence in their journals: I would give my design number ____ the name ___________ because _______________________________.

Extension activities

- Have students create a bulletin board collage with their designs.
- Figures 1–4 in the exploration activity used the standard right angle between the x and y axis. Have students construct a similar figure when the x and y axis creates an acute angle. Instruct them to number this figure "6," sign it, and turn it in.
- Tell students to construct a similar figure when the x and y axis creates an obtuse angle, number it "7," sign it, and turn it in.
- Challenge students to follow this method to construct a 3-dimensional figure.

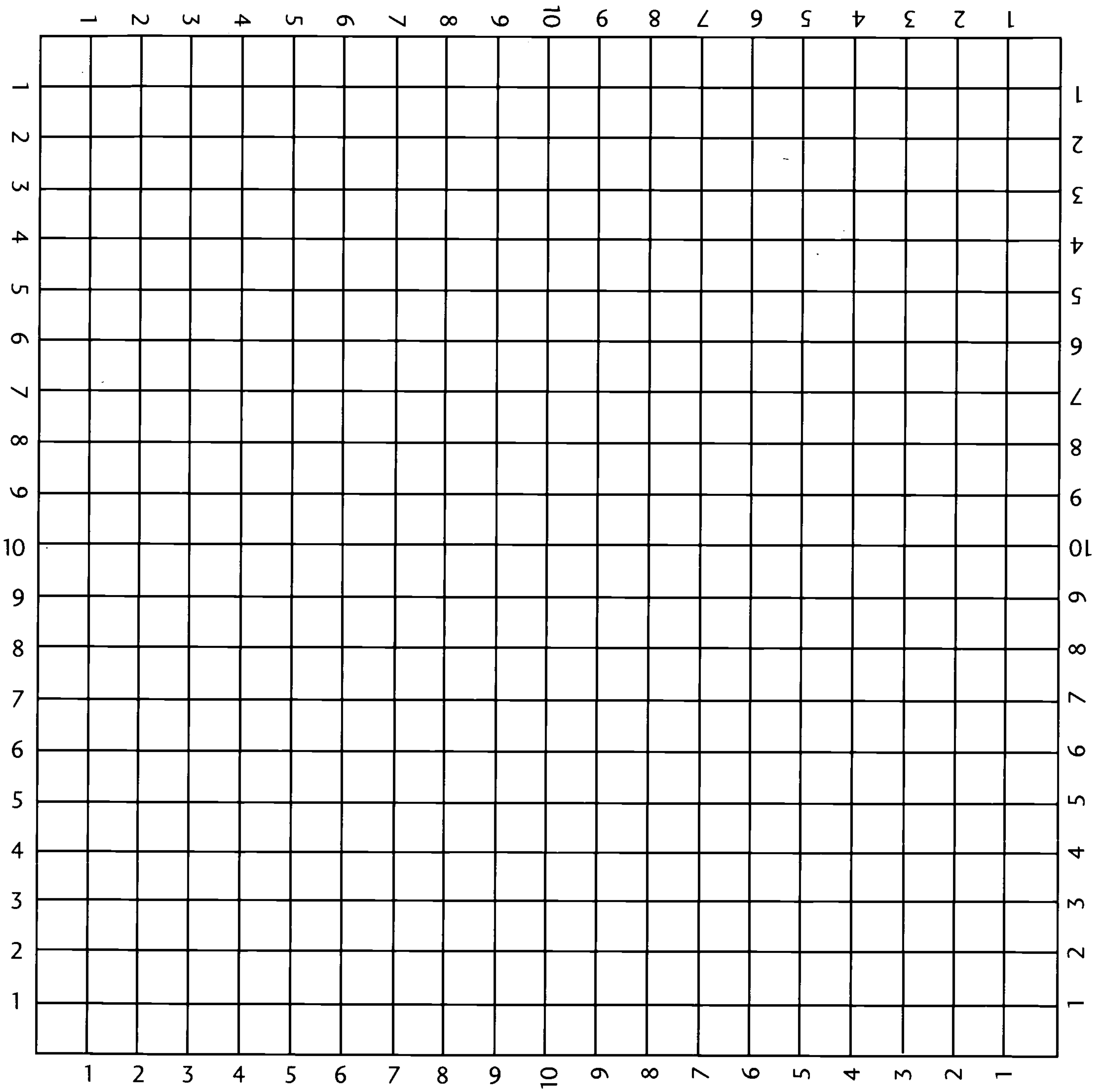
Connections to other subjects

Art. Have students explore the creation of curves with straight line segments in different mediums (e.g., watercolor, charcoal) and with different subjects (e.g., still life, landscapes).

Home connections

Tell students to construct a figure similar to a design created in the exploration activity, using board, nails, and string.

20 x 20 Paper



Plotting Sets

Instructions

Connect the following sets with line segments in each color designated. Then, record your observations about the figures created. Plot the sets in the appropriate corners of the 20 x 20 sheet, as labeled.

	Right	Up
1. Red		
	1	5
	2	4
	3	3
	4	2
	5	1

	Right	Up
2. Blue		
	2	10
	4	8
	6	6
	8	4
	10	2

	Right	Up
3. Green		
	1	10
	2	8
	3	6
	4	4
	5	2

	Right	Up
4. Purple		
	10	1
	8	2
	6	3
	4	4
	2	5

Lines and Dimensions

About this learning activity

Students will use geoboards to create 2-dimensional figures, and toothpicks and miniature marshmallows to create 3-dimensional figures.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II ○	IV P	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Geoboards
- Rubber bands
- 2-D and 3-D Worksheets
- Toothpicks and miniature marshmallows (Alternative: Gumdrops, pipe cleaners, or straws and tape can be substituted.)
- Cube objects (e.g., pyramids, cones, rectangular prisms, cylinders, diamonds, spheres)
- Rulers
- Protractors

Engagement activity

1. Have students begin the journal assignment.

2. Stop them after about 5 minutes and start a discussion about lines. Discuss the concepts of ray, segment, and line.

3. Discuss perpendicular, parallel, and intersecting lines.

4. Explain to students that they will use these line concepts in making 2- and 3-dimensional figures.

5. Let students finish their journal entries.

Exploration activity

1. Distribute geoboards, rubber bands, and Worksheets. Instruct students to create the 2-dimensional figures listed (problems 1–9) on the geoboards. Tell students that they may want to use rulers and protractors to measure lines and angles.

2. Explain that students are to use the figures they create to record the requested data (left column) on their Worksheets.

3. Lead a class discussion that helps students identify what they discovered during the exercise.

4. Display some examples of 3-dimensional figures. Let students name each object while you distribute the toothpicks and marshmallows.

5. Instruct students to finish the Worksheets by constructing the 3-dimensional figures with the toothpicks and marshmallows, and using the figures to record the data requested (i.e., number of vertices, edges, and faces).

6. If students are unable to construct a cone, cylinder, or sphere, suggest that they use the sample objects instead.

7. Help student discuss their findings. Finish the discussion with these questions:

   ? For which 2 figures was 0 recorded in all of the categories on the Worksheet? Why?

   ? Why is it important to have a working knowledge of the definitions of geometric terms like face, vertex, acute, and angle?

Evaluation

Grade the Worksheets. Answers can be found on pp. 194–195.

Journal assignment
Have students answer these questions in their journals:

? What do you know about lines?

? Where do you find different types of lines?

Extension activity

☼ Help students study volume, surface area, and object views (i.e., top, side, front, back).

Connections to other subjects

Art. Have students identify the use of 3-dimensional objects in modern art.

Home connections

Have students find each of the 3-dimensional forms studied in class in objects at home. Instruct them to make a list of the forms and where they found them.

Resources for teachers

Math in the Real World of Design and Art by Shirley Cook (Incentive Publications, 1996)

2-D Worksheet

Name ______________________

2-Dimensional Objects

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Name of figure									
Number of line segments									
= or ≠ lines (congruent)									
Parallel lines (yes or no)									
Number of vertices									
Number of right angles									
Number of obtuse angles									
Number of acute angles									

3-D Worksheet

Name ______________________________

3-Dimensional Objects

	Number of vertices	Number of edges	Number of faces
Cube			
Pyramid			
Cone			
Rectangular Prism			
Cylinder			
Diamond			
Sphere			

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2-D Worksheet Answers

2-Dimensional Objects

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Name of figure	square	rectangle	equilateral triangle	scalene triangle	isosceles triangle	circle	parallelo-gram	parallelo-gram	hexagon
Number of line segments	4	4	3	3	3	0	4	4	6
= or ≠ lines (congruent)	all =	opposite sides are =	all =	≠	2 =	0	opposite sides are =	opposite sides are =	6 =
Parallel lines (yes or no)	yes	opposite sides are =	no	no	no	0	opposite sides are parallel	opposite sides are parallel	opposite sides are parallel
Number of vertices	4	4	3	3	3	0	5	5	6
Number of right angles	4	4	0	0	1	0	0	0	0
Number of obtuse angles	0	0	0	1	0	0	2	2	6
Number of acute angles	0	0	3	2	2	0	2	2	0

209 210

3-D Worksheet Answers

3-Dimensional Objects

	Number of vertices	Number of edges	Number of faces
Cube	8	12	6
Pyramid	5	8	5
Cone	1	0	1
Rectangular Prism	8	12	6
Cylinder	0	0	2
Diamond	6	12	8
Sphere	0	0	0

Fun with Reflections, Translations, Rotations, and Stretches

About this learning activity

Students will make potato stamps for use in creating reflections, translations, rotations, and stretches.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II ○	IV P	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Potatoes (1 per student)
- Paint (tempera works best)
- Paint brushes (1 per color)
- Plastic containers (to hold paints)
- Sharp knife (to halve potatoes)
- Plastic knives (1 per student)
- Newspaper
- Paper towels
- Circle Rotation worksheets
- Translation worksheets
- Stretching worksheets
- Unlined paper

Engagement activity

Tell students that for the next 2 days they will be making potato stamps and experimenting with them to learn about rotations, translations, reflections, and stretches of geometric figures. Ask students whether they have heard these terms, and help them define and discuss them.

Exploration activities

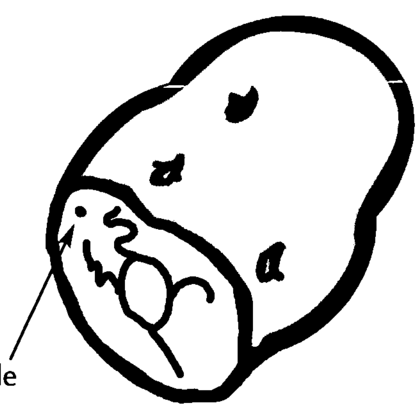
Day 1: Rotation and translation

1. Cut half of your potatoes into halves. (Save the remaining potatoes for Day 2.)

2. Distribute a plastic knife and potato half to each student. Instruct students to carve a simple geometric figure into their potatoes, according to these guidelines:

    - Figures must be carved about a centimeter deep.

    - Create a geometric figure that is different at all corners. It can be a shape or an irregular figure, as long as it is distinguishable at the corners.



Hole

3. Have students make a hole over the top of their figures, as shown in the illustration. This designates the top of the figure and will be helpful in conducting rotation and translation.

4. Instruct students to wipe their potatoes with a paper towel to remove excess water. Explain that painting will be difficult if they miss this step.

5. Have students apply 1 color of paint and make a practice stamp on a piece of scrap paper.

6. Distribute the Circle Rotation worksheets. Ask students to think about the rotation around a circle. (If students experience difficulty, suggest they think of it as a clock.) Then, explain that they will be slowly rotating their figures all the way around the circle on the worksheet. Have them physically rotate their potatoes in the air to demonstrate their understanding. Remind students to hold their figures in the "upright" position (i.e., with the hole designating the top of the figure at the top of the circle) **when they begin**.

7. Explain that they need to **plan** where they will place their potatoes before stamping so that the figure is stamped at 90°, 180°, 270°, and 360° and there is equal spacing between each image.

8. After confirming understanding, let students complete their circle rotations.

9. Facilitate a class discussion with these and other questions:

   - ? What happened to the positioning of the figure as you stamped around the circle? (Answer: It rotated.)
   - ? If you were given a picture of your figure at 75°, how would you be able to identify that it was shown at 75°?
   - ? Were you able to leave your hand in a comfortable position while stamping around the circle? Why or why not? (Answer: They had to twist their hands to reach 180°, then turn their hands to reach 270°.)

10. Distribute copies of the Translation worksheet. Have students stamp their figures on line 1, then have them slide the figures to the next line and stamp there.

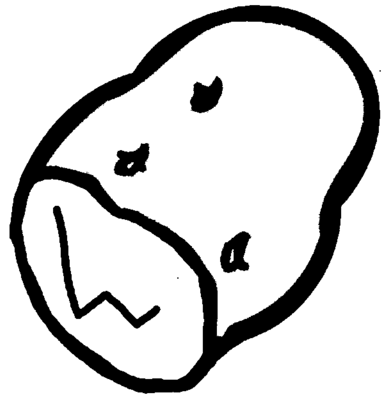
11. Tell students to continue this process until all 6 lines have been stamped. Then, explain that translation is simply sliding the figure.

Day 2: Reflection and stretching

1. Halve the remaining potatoes and distribute the halves and plastic knives.

2. Instruct students to draw another geometric figure to carve in their potatoes.

3. Have students follow the instructions in step 2 from Day 1, **except** they are to **carve only half** of their images into the potatoes (see illustration).



4. Have students wipe their potatoes with a paper towel to remove excess moisture, then brush them with paint and test their stamps on plain white paper.

5. Instruct students to draw the other half of their figures on their papers.

6. Explain to students that drawing their stamped figures was equivalent to making reflections across a line of symmetry. Further explain that the place where the stamp and the drawing meet is called the line of symmetry, then have students draw this line on their papers.

7. Have students switch stamps and repeat the activity, i.e., draw a reflection and line of symmetry for their partners' stamps.

8. Distribute the Stretching worksheets. Instruct students to stamp their papers in the top left corner of the worksheet (section #1).

9. Have each student calculate the approximate area of his/her stamp and record it in the left margin beside section #1. (Answer: Count the number of squares the figure covers.)

10. Instruct the students to stamp 2 figures in the upper right corner (section #2). Tell them to stamp the figures as close together as possible without overlapping them.

11. Have students calculate and record the approximate area covered by **both** figures in section #2.

12. Instruct the students to stamp 3 figures in the section marked #3 (across the bottom half of the page). Tell them to stamp the figures as close together as possible without overlapping them.

13. Have students calculate and record the approximate area covered by **all three** figures in section #3.

14. Explain to students that they have been "stretching" their geometric figures. Help them recognize that the areas they calculated should have doubled (section #2), then tripled (section #3).

15. Facilitate a class discussion of reflection and stretching, using these and other questions:

    - What objects have symmetry and can be reflected? (Answers include hearts, equilateral triangles, isosceles triangles, squares, rectangles, diamonds.)
    - How many lines of symmetry can be found in these different figures?
    - Could you use your stamps from Day 1 to make a reflection? (Answer: No, because the figure must be 2-sided to make a true reflection.)
    - Why did the **areas** double and triple in the stretching activity? (Answer: You are doubling and tripling the **figure**.)
    - How is this like multiplication? (Answer: Multiplying the first number by 2, then by 3, obtains the same results as counting the graph paper squares in sections #2 and #3, respectively.)
    - If you draw a rectangle 6 square feet in size, then double it, how large will the rectangle's area become? (Answer: 12 sq ft)
    - If I decided to double all 4 sides of a rectangle, would the rectangle's area double too? Explain. (Answer: No, it would quadruple.) *Hint: Demonstrate this problem on the board to help students grasp the answer.*

Real-life applications

Discuss applications of rotations, translations, reflections, and stretches of geometric figures in architecture and engineering. Examples include calculating areas for floor coverings, siding, and painting in building construction and similar interior design applications.

Evaluation

- Distribute additional worksheets and have students complete the exploration activities again for a grade.

- Distribute pictures of different figures and have students find the line of symmetry and reflection for each. Note: Be sure to provide some figures that do not have these attributes.

- Have students examine each capital letter of the alphabet for symmetry, then draw a reflection of each letter in their names.

Journal assignment
Have students write some or all of these assignments in their journals:

- Describe some common examples of rotations and translations.

- Discuss the number of lines of symmetry in a circle. (Answer: Infinite)

- Identify which reflections of the letters in the alphabet would look the same and which would look different, then explain why. (Answer: All with symmetry will look the same, i.e., A, B, C, D, E, H, I, M, O, T, U, V, W, X, and Y.)

Home connections

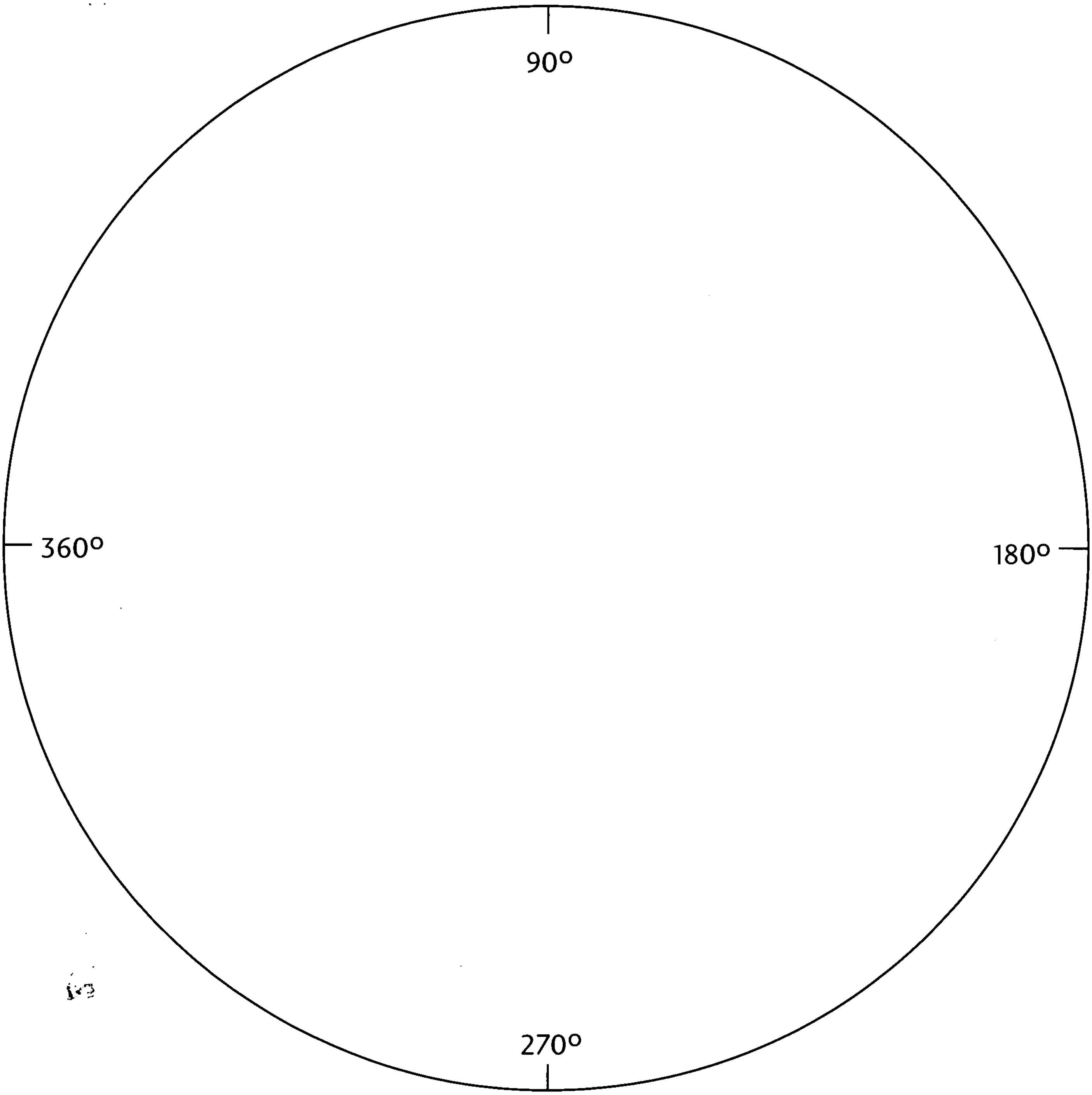
Have students look for rotations, translations, reflections, and stretches in building architecture and in common objects.

Resources for teachers

Triangles and Beyond by Britannica Mathematics System, Mathematics in Context, Triangles and Beyond (Encyclopedia Britannica Educational Corporation, 1996)

Circle Rotation Worksheet

Name ____________________



Translation Worksheet

Name ______________________

(1)

(2)

(3)

(4)

(5)

(6)

220

221

Stretching Worksheet

Name ______________________________

(1)

(2)

(3)

Screen Saver

About this learning activity

Students will use their knowledge of geometry to design screen savers.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○ III ○ V ○ VII ○
II ○ IV P VI ○ VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Mirrors (or other instruments for reflections)
- Miras (or other transparent geometry tools—optional)
- Drawing paper (i.e., $8^1/_2$ x 11-in plain paper or $^1/_2$-, $^3/_4$-, or 1-in grid paper)
- Markers (or colored pencils)
- Rulers
- Protractors
- Screen Saver worksheets
- Tangram shapes or pattern blocks (optional)
- Geometric shape template (optional)

Engagement activity

1. Display a simple drawing. Facilitate a class discussion with these and other questions:

    ? What will the picture look like if I hold it in front of a mirror?

    ? How about if I turn it upside down?

    ? What happens to the picture if I take 2 steps to my left?

2. Discuss the definitions of symmetry, rotation (turn), translation (slide), reflection (flip), tessellation, and dilation (stretch). If desired, show pictures that illustrate each term (see the Teacher Resources section).

3. Talk about computer screen savers. If possible, demonstrate a few to the class. Ask students to describe screen savers they have seen (e.g., at the library, at home, at parents' offices). Be sure to discuss how screen savers change (e.g., scroll, bounce).

Exploration activity

1. Distribute several Screen Saver worksheets to each student. Ask students to design an original screen saver for a computer, according to the following instructions:

    - Create an original or start screen.

    - Create at least 5 additional screens. Note: This task requires at least 6 screens.

- At the bottom of each sheet, write a caption that describes how the picture transitions from 1 screen to the next—by rotation, reflection, translation, or dilation. (Transition methods may be used more than once, but each must be used **at least once**.)
- Use of rulers is mandatory.

Hint: You may wish to set other requirements for the students. Examples:

- *Do not use only angles that are 90° or 180°.*
- *Use triangles.*
- *Squares and rectangles are okay, but use other quadrilaterals too.*
- *Use circles.*

2. Encourage students to use tools (e.g., mirrors, Miras, protractors) to enhance creativity.
3. Explain that students should practice on plain paper, then put their final designs on the Screen Saver worksheets.
4. Facilitate a class discussion with these questions:

   ? Which is easiest to design: rotation, reflection, or translation?

   ? Explain what it means to dilate or stretch a figure.

   ? How many lines of symmetry can be found in your original screen?

   ? How many degrees was your turn (of rotation)?

   ? How many flips would be required to bring the original image back to the screen?

Explanation

Solutions will vary, but make sure students follow the parameters set. For younger grades, you may wish to provide the start-up drawing and set fewer parameters.

Evaluation

- Observe students' knowledge and use of the activity's vocabulary.
- Observe students' use of tools like protractors and rulers.
- Give students a series of pictures and have them determine the different processes that resulted in the succeeding images.

Journal assignment
Have students explain the terms rotation, translation, reflection, and dilation in their own words.

Extension activity

- Expand the exploration activity by increasing the parameters and adding 3-dimensional figures (e.g., prisms, spheres) and pentagons, hexagons, or other polygons.

Connections to other subjects

Computer Science. Have students study screen savers, including writing a software company to learn some *behind the scenes* information about creating screen savers.

Art. Have students observe pictures and structures to identify examples of symmetry, rotations, translations, and dilations.

Home connections

Ask family members to help students identify different designs and tessellations on floor tiles, in carpets, and in buildings.

Resources for teachers

Designs for Coloring: More Geometrics by Ruth Heller (Grosset and Dunlap, 1991)
Geometrical Design Coloring Book by Spyros Horemis (Dover Publications, 1973)
Slides, Flips and Turns by Luis Kroner (Dale Seymour Publications, 1994)

Screen Saver Worksheet

Name ______________________________

Screen Number ______

Notes

Fun with Math:
Real-Life Problem Solving for Grades 4–8

Strand V: Algebra

Learning Activities **Page**

Notes

Distribution Breakdown

About this learning activity

Students will use manipulatives to recognize the distributive property of multiplication.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V P	VII ○
II ○	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Hint: Before starting the exploration activity, make several pairs of index cards with 2 sides of the distributive property split on the 2 cards. For example, Problems #1 and #2 from Step 1 of the engagement activity make 2 sides of the distributive property. You would write 1 problem on each card to show the distributive property split on 2 cards.

Materials

- Manipulatives (e.g., beans, buttons, bingo chips)
- Index cards

Engagement activity

1. Write these problems on the chalkboard:

Problem #1	Problem #2
3 x 14 =	3(10 + 4) = (3 x 10) + (3 x 4) =

2. Ask a student to solve the first problem while you solve the second problem.

3. Help students examine the difference(s) in the problem-solving methods used. Discuss the second problem to show that the 3 is **distributed** over the 10 and the 4, but the answers to both problems are the same. Use these and other questions in the discussion:

   ? Is there any difference in the answer?

   ? Which problem is easier to solve? Why?

4. Distribute a supply of manipulatives to each student. Have students illustrate both problems with the manipulatives.

Exploration activity

1. Distribute the manipulatives again, if necessary. Have students illustrate the following problems:

2 x 5 = 10	2(3 + 2) = 10
3 x 10 = 30	3(6 + 4) = 30
4 x 7 = 28	4(5 + 2) = 28

2. Next, distribute an index card to each student. Be sure that both cards in each distributive pair are distributed.

3. Have each student illustrate his/her card with the manipulatives.

4. Then, instruct students to search the other illustrations in the room, cards in hands, for a match.

5. Help students recognize that they have been working with the **distributive property** for multiplication:

   If the sum of 2 numbers is multiplied by a third number, the answer will be the same whether the numbers are added together, then multiplied or each is multiplied by the third number, then added.

Real-life applications

Help students apply the concept of distributive property to their daily lives with this problem:

> *Sixteen students are going to the movies. Tickets cost $6.50 each. Use distributive property to simplify this problem.* [Answer: (16 x $6.00) + (16 x $.50) = $96 + $8 = $104.]

Evaluation

Ask the students to complete the Distributive Property Worksheet (p. 217).

Journal assignment
Have students complete this sentence in their journals: When I am asked to take the sum of 2 numbers and multiply by another number, I can simplify the problem-solving process by . . .

Extension activities

☼ Instruct students to write this problem in distributive property form:

> *Carey works part time at the soda shop. She earns $24.50 per day. How much will she earn in 5 days?* [Answer: 5($24 + $.50) = ____.]

☼ Have students apply the distributive property to planning homework assignments with this problem:

If Joe has to read 8 pages of Social Studies and 6 pages of English each evening, how many pages does he have to read in 4 days? [Answer: 4(6 + 8) is the same as (4 x 6) + (4 x 8).]

Home connections

Ask the students to explain the distributive property to family members.

Resources for teachers

When Are We Ever Gonna Have to Use This? by Hal Saunders (Dale Seymour Publications)
Mathercise by Michael Serra (Key Curriculum Press, 1992)

Computer programs:
- *Everyday Math*
- *Super Solvers Outnumbered!* (The Learning Company)
- *Math Blaster Mystery* (Davidson)

Distributive Property Worksheet

Name ______________________________

4 x 38	=	(4 x 30)	+	(4 x ?)		
7 x 940	=	(? x 900)	+	(7 x ?)		
10 x 483	=	(10 x 400)	+	(10 x 80)	+	(10 x ?)
9 x 794	=	(? x 700)	+	(9 x ?)	+	(? x ?)
5 x 143	=	(5 x 100)	+	(? x ?)	+	(? x ?)

More Distribution

About this learning activity

Students will use base ten rods and unit cubes to illustrate the distributive property of multiplication.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V P	VII ○
II ○	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Base ten rods
- Unit cubes
- More Distribution worksheets

Engagement activity

Distribute the rods and cubes. List several numbers on the board and instruct students to use the materials provided to create them.

Exploration activity

1. Explain to students that you want to multiply 13 by 3. Then, illustrate 13 with a base ten rod and 3 unit cubes.

2. To multiply by 3, illustrate 3 sets of this pattern, explaining that the solution is 3 base ten rods and 9 unit cubes or 39.

3. Demonstrate that the solution can be written (3 x 10) + (3 x 3) or 3(10 + 3).

4. Illustrate the problem 23 x 2 with 2 base ten rods and 3 unit cubes.

5. Show 2 sets of this pattern for multiplying by 2, explaining that the solution is 4 base ten rods and 6 cubes or 46.

6. Demonstrate that the solution can be written (2 x 20) + (2 x 3) or 2(20 + 3).

7. Illustrate the problem *26 x 3* with 2 base ten rods and 6 unit cubes.

8. Show 3 sets, explaining that the solution is 6 base ten rods and 18 cubes. Further explain that 10 of the cubes can be replaced with a base ten rod, leaving 7 rods and 8 unit cubes or 87.

9. Demonstrate that the solution can be written (3 x 20) + (3 x 6) or 3(20 + 6).

10. Next, explain that you want to multiply 49 by 3. Help students recognize that you could illustrate 49 as 4 base ten rods and 9 unit cubes, use 3 sets of this pattern, and replace all groups of 10 cubes with base ten rods. Ask if anyone can think of an easier method.

11. If students need help, suggest that they think about using estimation. Help students recognize that they could round 49 up to 50, illustrate 50 with 5 base ten rods, and place 1 unit cube off to the side to represent the difference between 50 and 49.

12. Demonstrate this method and show 3 sets. Point out that there are 15 base ten rods, with 3 unit cubes sitting to the side, making the solution 150 – 3 or 147.

13. Show that the solution can be written (3 x 50) – (3 x 1) or 3(50 – 1).

14. Ask students to name the multiplication principle you have been using for this exercise. (Answer: Distributive property)

Hint: You may wish to help student volunteers illustrate problems for the class after you demonstrate the first problem.

15. Instruct students to complete the More Distribution worksheet, using the same techniques demonstrated with the previous problems. Encourage them to try writing the problem using the distributive property **before** illustrating it with the base ten rods and unit cubes.

16. Circulate to check students' progress, providing guidance as needed.

Explanation

This lesson is intended to allow students to practice using the distributive property. It is not intended to be an entire treatment of the principle.

Evaluation

Grade students' More Distribution worksheets.

Journal assignment
Ask students to explain how 124 x 3 can be solved without paper by using the distributive property.

Extension activities

- Conduct a "Mental Math Multiplication Bee." (Instructions: All students stand. One at a time, each student is given a problem to solve in his/her head. Students sit down if they miss a problem. The last person standing wins.)

☼ Ask students to use the distributive property in solving this problem:

A family of 5 goes to a restaurant for dinner. Everyone has the $6 buffet and $1 drinks. Show 2 ways to calculate the cost of the family's meal.
[Answers: 5($6 + $1) = $30 + 5 or 5 x $7 = $35]

Home connections

Ask family members to help students do mental multiplication during a trip to the grocery store. For example:

- 2 loaves of bread @ $1.09 totals _____? [Answer: 2($1 + $.09)]
- 3 cans of soup @ $.89 totals ______? [Answer: 3($.09 - $.01)]

Resources for teachers

Base 10 Block Activities (Nasco Math)
Mental Math in the Middle Grades by J. Hope, B. Reys, and R. Reys (Dale Seymour Publications)

More Distribution

Name ______________________________

Record each problem in distributive property form before solving it.

1. 12 x 4
2. 13 x 5
3. 18 x 4
4. 51 x 6
5. 89 x 6
6. 34 x 3
7. 68 x 3
8. 92 x 6
9. 29 x 6
10. 81 x 7
11. 73 x 7
12. 49 x 6

One Check or Two?

About this learning activity

Students will choose items from a menu, then use the distributive property to calculate the charges.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V P	VII ○
II ○	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Pencils and paper
- Wallets containing play money (If not available, copy and cut out the play money on p. 228.)
- Servers Order Forms
- Jo-Jo's Menus
- *Wait To Be Seated* sign
- Calculators (1 for each check-out area)

Note: Before beginning this lesson, create a restaurant in your room by arranging desks into pairs, facing each other (like restaurant booths). Set up a check-out area in each corner of the classroom. Then, prepare the wallets, making sure that each contains an amount and variety of bills and coins to pay for 2 of **any** menu selection.

Engagement activity

1. Hang the *Wait to Be Seated* sign outside the classroom door.

2. As students enter the classroom, hand the first 2 *customers* a menu and a wallet. Hand the third who enters an order form and explain, "You're the server." Send all 3 students to the same *booth*.

3. Repeat Step 1 until all students have entered and found their *booths*.

4. Instruct the customers to study the menu and decide what to order, following the instructions on the menu. Instruct the servers to look over their order forms and prepare to take their customers' orders.

5. Explain that you are functioning as the restaurant manager today, and ask students to wait for your signal to begin taking orders.

Exploration activity

1. Instruct servers to start taking their customers' orders. Remind students that the instructions on the menus and order forms are to be followed.

2. Circulate among the booths to answer questions and observe transactions.

3. Once all order forms have been completed and the money collected, choose a group to go to the board. Instruct the server to write the left side of his/her booth's equation. Example: 2 x $4.15.

4. Instruct 1 of the customers to write the right side of the booth's equation. Example: 2 x 4 + 2 x 15.

5. Tell the other customer to solve the equation.

   Example: $8.30 = 8 + $.30
   $8.30 = 8.30

6. Repeat Steps 3 through 5 with a few more groups. Then, instruct the remaining groups to complete these steps at their booths.

7. Facilitate a class discussion with these and other questions:

   ? Why do you get the same answers whether you multiply the total by 2 or double dollars and double cents, then add?

   ? Would this work with 3 customers and 1 server instead of 2 customers? With 4 customers and 1 server?

   ? Can you name this property? (Answer: Distributive property.)

   ? Can you write it algebraically?

8. If time allows, regroup customers and servers for a 3-to-1 ratio and repeat the activity.

Evaluation

Observe as servers collect for their orders. Confirm that they collect the correct amounts.

Home connections

Send a letter to family members that explains the exploration activity and ask that students be allowed to handle the calculations and money in similar real-life situations for practice.

Order Form

Instructions

1. Take your customers' orders.

2. Go to a check-out area and calculate your customers' charges. You may use a calculator. Note: Multiply the price of the item ordered by the number of customers at your booth, since they ordered the same thing.

3. Write the total charges at the bottom of the order form.

4. Return to the booth and collect the exact amount owed. (Sorry, no tip today.)

Menu Item	Price
Cousin Priscilla's plain cheeseburger, fries, and medium Sprite	$1.19
Uncle Rick's garbage burger (Half-pound burger stacked high with 3 cheeses, lettuce, and tomato.) Served with a HUGE basket of fries and onion rings. Wash it down with an extra large soda of your choice.	$4.15
Aunt Edna's garden salad with diet cola	$2.22
Papaw's perfect pizza with large soda	$3.20
TOTAL:	

Jo-Jo's Menu

Instructions

1. For purposes of this exercise, you and your lunch partner must order the same items from the menu. Review the menu and agree on a common order.

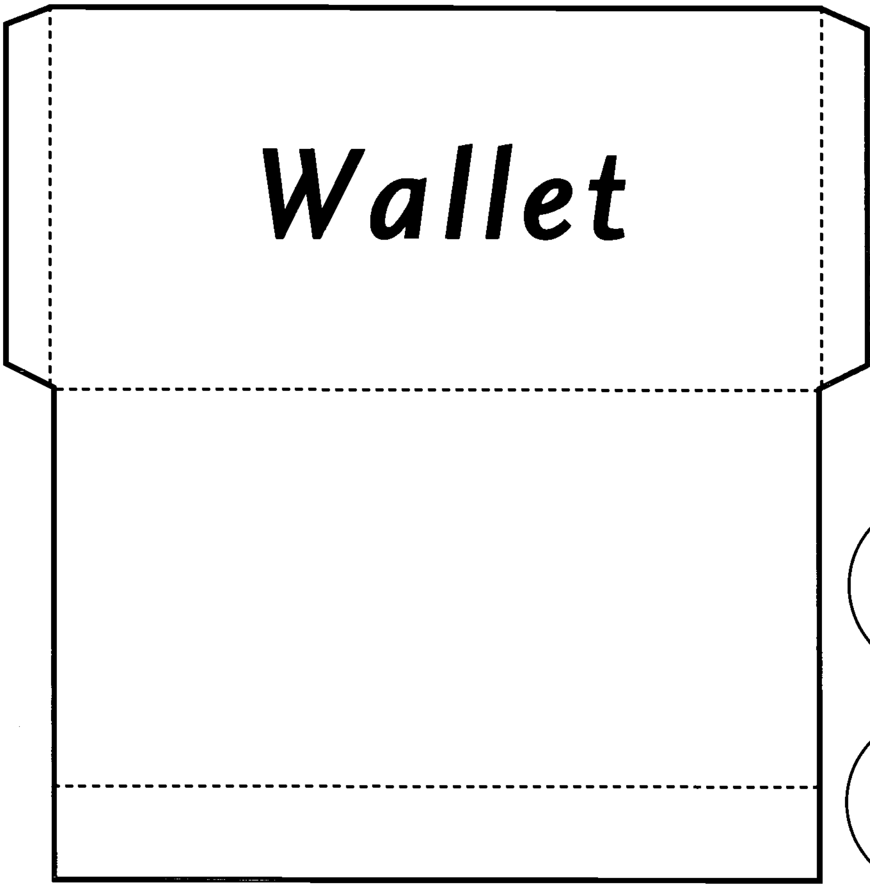
2. Before your server returns to bring you the check, calculate how much you owe. Note: Lucky you! This restaurant doesn't charge tax or tips.

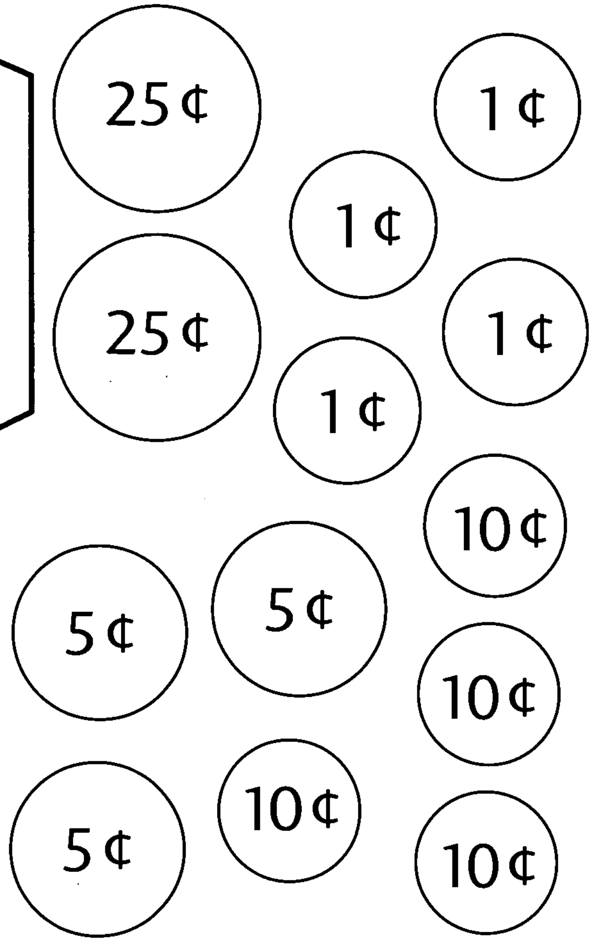
3. Remove the exact amount owed (dollars and cents) from your wallet.

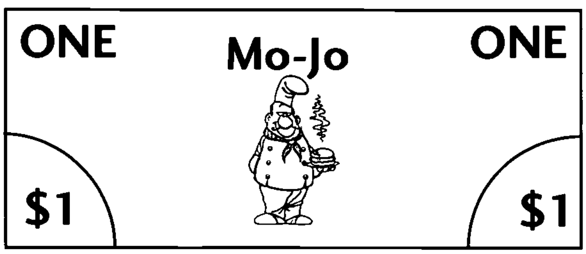
4. Give the exact amount to your server.

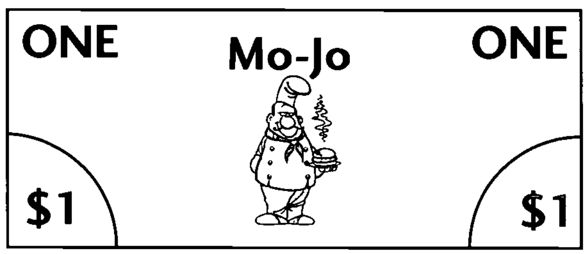
Menu Item	Price
Cousin Priscilla's plain cheeseburger, fries, and medium Sprite	$1.19
Uncle Rick's garbage burger (Half-pound burger stacked high with 3 cheeses, lettuce, and tomato.) Served with a HUGE basket of fries and onion rings. Wash it down with an extra large soda of your choice.	$4.15
Aunt Edna's garden salad with diet cola	$2.22
Papaw's perfect pizza with large soda	$3.20

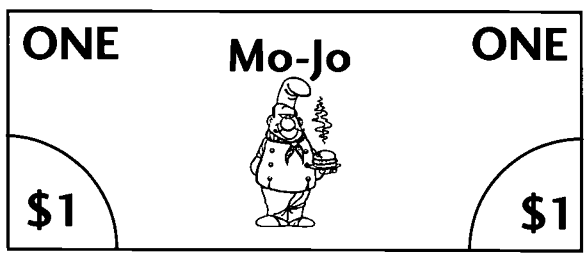
TEMPLATES FOR WALLET AND MONEY











It's In the Bag

About this learning activity

Students will use manipulatives to change simple word sentences into algebraic equations.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V P	VII ○
II ○	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Paper lunch bags
- Counters (e.g., cm cubes)
- Index cards (to create number and +, –, and = cards)
- Paper and pencils

> Note: Prior to starting the lesson, prepare a bag set for every 3 students. Each bag set will contain counters inside a sealed (twisted shut) paper bag, 2 number cards, an operation card (+ or –), and an equal card (=).
>
> *Sample Bag Set:*
> - 7 counters inside a bag that is twisted shut
> - 2 number cards (5 and 12)
> - Operation card (+)
> - Equal card (=)
>
> Possible equations: $12 + 5 = \square$; $5 + \square = 12$
>
> Impossible: $12 + \square = 5$ (negative outcome)

Engagement activity

1. Give each team of 3 students a bag set. Instruct students not to open the bags until told to do so.
2. Tell teams to predict the number of counters in their bags **without opening them.** Explain that they must record their predictions on a sheet of paper and show their work.
3. Have teams exchange bags and repeat Step 2.
4. Repeat, as desired.

Exploration activity

1. Using the sample bag (unopened), write possible true number sentences on the board. Draw a picture of the bag to represent the bag portion of the sentence, since you don't know how many counters are in the bag.

2. Have teams exchange bag sets again and write true number sentences (including bag pictures) for their sets.

3. Discuss the disadvantages of having to draw a picture each time you want to show the contents of the bag in a sentence. Ask students what else they could use. (Possible answers: A box, a line segment, the word *bag*) List their suggestions down a side of the board.

4. Write 1 of the team's sentences on the board with the picture of the bag representing its contents. Rewrite it using as many of the suggestions as possible. *(Hint: If no one suggests using a letter to represent the contents of the bag, introduce that choice after using the word* bag.*)*

5. Instruct teams to carefully open their bags, take out the counters, and check their work.

6. Facilitate a class discussion with these and other questions:

   ? How many teams accurately predicted the number of counters in the bags?

   ? What prediction methods did they use?

   ? Did the method used affect the accuracy of the prediction? Why or why not?'

7. Instruct teams to twist shut the now-empty bags.

8. Have teams **trade operation cards** and write a true sentence with the bag set they have now—2 number cards, 2 symbols, and an empty bag. Example: Bag + 5 = 12 might become 12 – 5 = Bag or 12 – 5 = B.

9. Encourage groups to use a letter to represent the bag portion of the true sentence.

10. Define the term *variable*, if students have not attempted to use the word correctly yet. Explain that students have been changing simple word problems into algebraic equations in this exercise.

Evaluation

Give students 5 simple word sentences and instruct them to change each into an algebraic equation.

Journal assignment
Have students explain the sentence: $n + 4 = 7$.

Paper Equations

About this learning activity

Students will use paper strips to represent, balance, and solve algebraic equations.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V P	VII ○
II ○	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Equal Sheets
- Construction-paper strips (1 x 9-in; 2 yellow, 4 blue, 2 red)
- Algebraic Equations worksheets

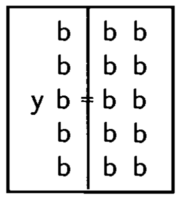
Engagement activity

Distribute 2 yellow, 4 blue, and 2 red construction-paper strips to each student. Instruct students to tear the yellow strips into 4 equal pieces, and the blue and red strips into 8 equal pieces each. Explain that for the upcoming exploration activity, the yellow pieces are the variables, the blue pieces are the numbers and add-ons, and the red pieces are those numbers we subtract.

Hint: Illustrated solutions to the sample problems can be found on p. 237.

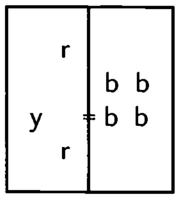
Exploration activity

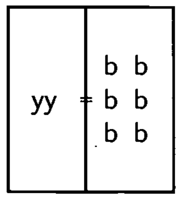
1. Distribute an Equal Sheet to each student. Demonstrate how to create a paper equation with these 3 problems:



$5 + n = 8$ (Lay 5 blue strips and 1 yellow strip on the left side, and 8 blue strips on the right.)

$x - 2 = 4$ (Lay 1 yellow strip and 2 red strips on the left, and 4 blue on the right.)





$2n = 6$ (Lay 2 yellow strips on the left and 6 blue strips on the right.)

2. Explain to students that solving Paper Equations requires following 3 basic rules:

#1 Whatever you remove or add to 1 side, you must add to or remove from the other side.
#2 A red strip and blue strip on the same side cancel each other, so both can be removed.
#3 When more than 1 variable strip is used, separate each side into equal patterns. Remove all extra patterns (keep only 1).

Write the rules on the board or distribute them in a handout.

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3. Help students use the 3 basic rules to work the sample equations with you. Start with $5 + n = 8$. Write it on the board and have students create a paper equation for it on their Equal Sheets.

4. Explain that the goal is to isolate the yellow strip (i.e., the variable). Helps students isolate the yellow strip with these questions:

   ? To make the yellow strip stand alone, what must we remove? (Answer: We must remove the blue strips from the left side.)

   ? To follow rule #1, what else must we remove? (Answer: We must remove a blue strip from the right side for every blue strip we remove from the left side.)

5. Have students carry out the solution steps discussed. Circulate to check their work.

6. Ask, "What does n equal?" (Answer: 3)

7. Have students create a paper equation for $x - 2 = 4$. Explain that the goal, again, is to isolate the yellow piece (variable). Facilitate the solution process with these questions:

   ? How can we isolate the variable (or yellow strip)? (Answer: We need to remove the red strips.)

   ? How do we follow the rules to do this? (Answer: We have to add a blue strip on the left side for every red strip on the left side. Then, we must add an equal number of blue strips to the right side.)

8. Have students carry out the solution steps discussed. Circulate to check their work.

9. Ask, "What does x equal?" (Answer: 6)

10. Have students create a paper equation for solving $2n = 6$. Facilitate the solution process with these questions:

    ? What's the first step? (Answer: We need to isolate the variable.)

    ? Since this problem has 2 yellow strips (variables), what rule do we need to follow? (Answer: Rule #3. We must separate each side into 2 equal patterns.)

? What pattern should we use? (Answer: The pattern is 1 yellow to 3 blue.)

? So what does n equal? (Answer: 3) Note: Have students carry out the solution steps discussed before asking this question.

? How would you solve $2n + 3 = 7$? (Answer: Before we separate the variables into patterns, we must isolate the variable. The pattern is 1 yellow to 2 blue.)

? So what does n equal? (Answer: 2)

11. Demonstrate each type of equation in this manner, then distribute the Algebraic Equations worksheet. Have students complete it **without using the manipulatives.**

Evaluation

Have students complete the worksheet without using the manipulatives. *(Hint: You may wish to let students use the manipulatives to check their answers.)*

Journal assignment
Instruct students to write a narrative that explains how to solve $2x + 4 = 10$.

Home connections

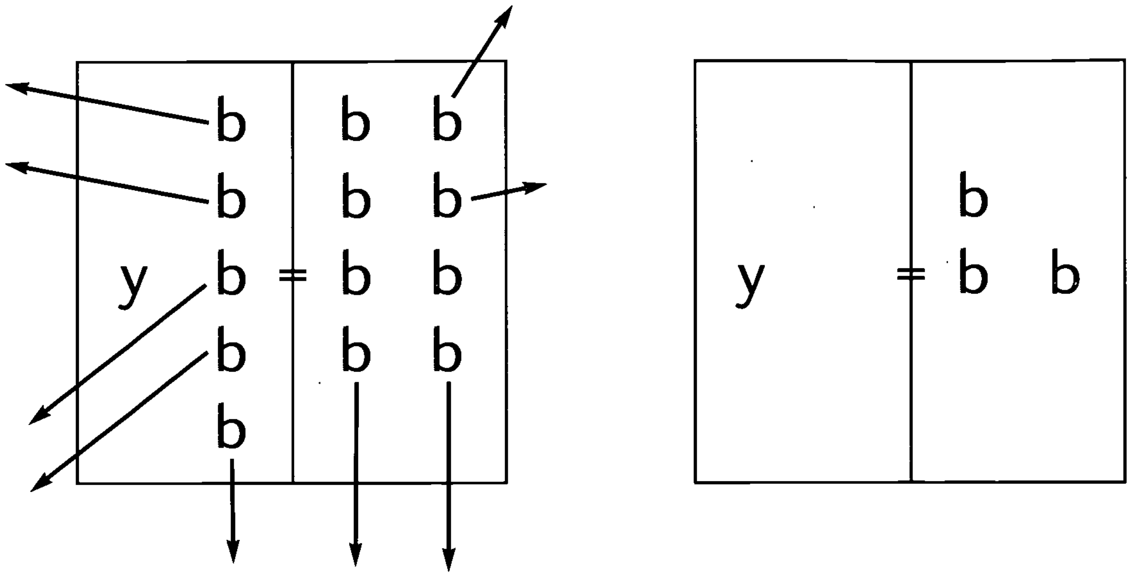
Tell students to locate and list opposite operations (e.g., on/off, in/out) at home.

Resources for teachers

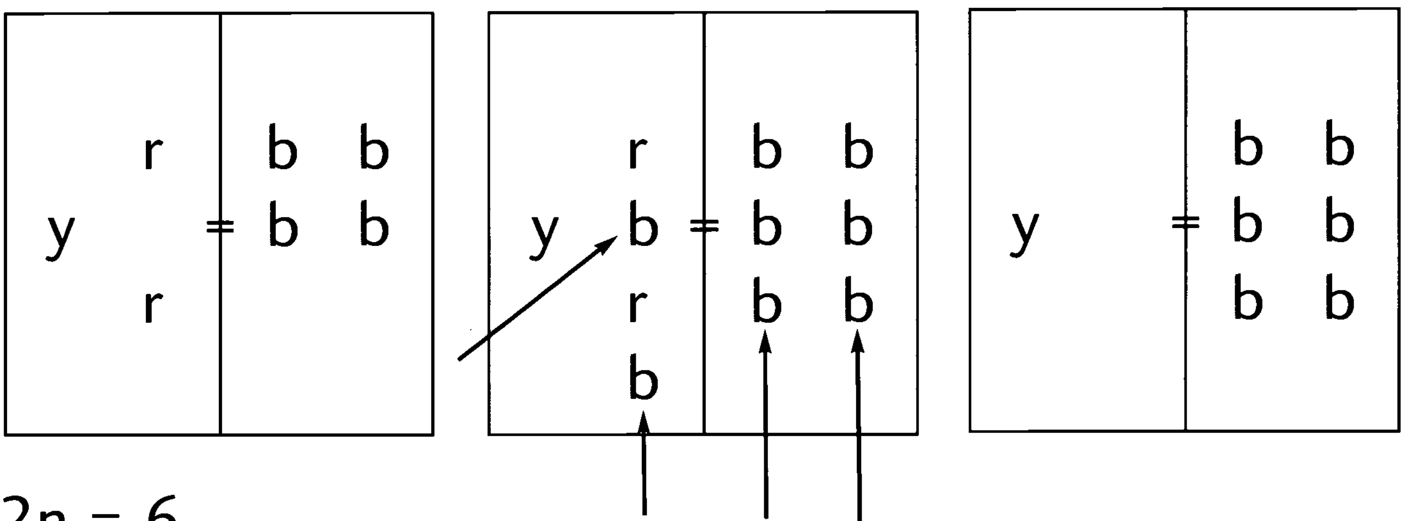
Algebra Tiles for the Overhead Projector by Hilde Howden (Cuisenaire Co.)
Algebra Domino Links (game by Nasco Math)

Solutions to Sample Problems

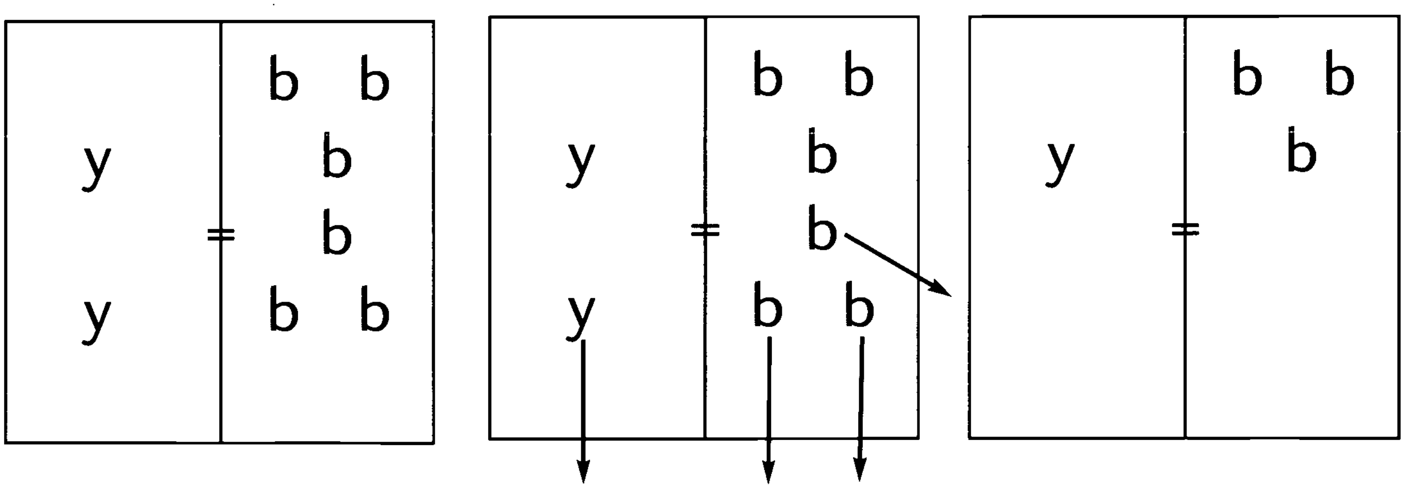
$5 + n = 8$



$x - 2 = 4$



$2n = 6$



Equal Sheet

$=$

Algebraic Equations

1. $4 + n = 7$	2. $x + 3 = 5$	3. $n + 3 = 10$
4. $n + 4 = 9$	5. $n + 7 = 12$	6. $5 + x = 11$
7. $g + 2 = 9$	8. $8 + c = 15$	9. $k + 6 = 14$
10. $n + 6 = 16$	11. $x + 9 = 15$	12. $n + 10 = 15$

1. $n - 2 = 8$	2. $x - 5 = 18$	3. $y - 6 = 12$
4. $n - 8 = 11$	5. $x - 6 = 2$	6. $n - 7 = 6$
7. $n - 3 = 4$	8. $x - 11 = 5$	9. $g - 6 = 10$
10. $n - 5 = 13$	11. $n - 3 = 25$	12. $x - 7 = 14$

1. $2n = 14$	2. $3y = 15$	3. $4n = 24$
4. $2n = 20$	5. $6x = 24$	6. $3y = 18$
7. $7n = 21$	8. $5x = 25$	9. $4x = 16$
10. $8n = 24$	11. $2y = 18$	12. $3n = 27$

1. $2n + 1 = 15$	2. $2x + 2 = 14$	3. $3x - 1 = 14$
4. $3n - 5 = 13$	5. $5n + 2 = 22$	6. $6x - 1 = 17$
7. $2x - 4 = 16$	8. $3x + 5 = 23$	9. $3x - 8 = 19$

Notes

Fun with Math: Real-Life Problem Solving for Grades 4–8

Strand VI: Measurement

Notes

Go the Distance!

About this learning activity

Students will rotate through a series of work stations to explore the concepts of perimeter, area, and volume.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III X	V ○	VII ○
II ○	IV ○	VI P	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Poster board
- Translucent (e.g., plastic, mylar) geometric shapes *(Hint: Cut shapes from plastic report covers.)*
- 1-cm graph paper (see p. 248)
- Stations worksheets
- Coffee can (empty and cleaned)
- Soup can (empty and cleaned)
- Small box
- Several cm cubes
- Go the Distance! worksheets
- Shoe box filled with candy treats
- Metric rulers
- Calculators

Engagement activity

Write this problem on the poster board before starting the activity:

> *You have been hired to prepare backgrounds for traffic signs. Signs come in the following shapes: octagon, rectangle, and circle. Backgrounds must be painted with yellow safety paint. A special reflective decal strip must be placed around the outside edge of each sign. How will you calculate the amount of yellow safety paint you will need to cover the sign backgrounds and the amount of decal tape you will need for the sign edges?*

Display the problem. Have students work in teams to discuss the answer, then share their conclusions. Note: You may wish to remind students that they cannot calculate the amounts since they do not know how many signs they need to make.

Exploration activity

1. Create the stations described below in various areas around the room.

   Area Station. Provide translucent (e.g., plastic, mylar) shapes on a table with a sheet of 1-cm graph paper. Post instructions that tell students to determine the area of each shape and record their findings on their Stations worksheets (see sample, p. 249).

   Perimeter Station. Provide a second set of the same shapes used in the Area Station and another sheet of graph paper. Post instructions that tell students to determine the distance around each figure (i.e., perimeter) and record their findings on their Stations worksheets.

   Volume Station. Provide the coffee can, soup can, and small box (or various sizes of empty cans and boxes) to represent solid figures, and several cm cubes. Post instructions that tell students to fill the containers with the cubes, count the number of cubes required, and record their findings on their worksheets.

Pencil/Paper Math Station. Provide a stack of Go the Distance! worksheets (1 for each student). Post instructions that tell students to take a worksheet back to their desks and compute the problems. Note: The worksheet contains the same problems the students worked in the previous 3 stations and formulas for calculating the answers.

Ruler Application Station. Place a shoe box filled with candy on top of a table. Tape the box lid shut tightly. Post instructions that tell the students to find and record the:

- Perimeter around the top of the box.
- Surface area of the faces of the box (combined).
- Volume of the box.

2. Instruct students to complete the **first 4 stations only**, then await further instructions.

3. When all students have completed the first 4 stations, facilitate a class discussion with these and other questions:

    - Compare the results of your Stations and Go the Distance! worksheets. What do you notice about the problems in each? (Answer: They are the same. The Stations worksheet uses a counting method to calculate the answers and the Go the Distance! worksheet uses formulas.)
    - Were your answers the same then? Why or why not?
    - Do you prefer counting or computing area? Why?
    - Do you prefer counting or computing perimeter? Why?
    - Why is counting volume with the cm cubes less accurate than computing it with a formula?
    - What happens to the area, perimeter, and volume of a figure as you increase the size of the figure?

4. Have students complete the final station, then discuss the results.

5. After students have completed the Ruler Application Station and/or calculated all the correct answers, open the box and distribute the candy. Note: This station should be completed last. You may wish to use it for evaluation purposes.

Real-life applications

Tell the students that they are going to buy carpeting for a living room that measures 15 x 15 ft. Explain that the store sells carpet by the square yard. Help students convert the measurements to determine the correct amount of carpet needed.

Evaluation

Grade students' calculations from the Ruler Application Station.

Have the students measure the length and width of a school hallway using a 12-in ruler. Then, ask them to find the area of the hallway in square yards. (The chart on p. 251 can be used for this evaluation activity.)

Journal assignment
Have students explain the difference between square units (area) and cubic units (volume) in their journals.

Extension activities

- Instruct groups of students to find the perimeter of the playground with a measuring wheel, then use their measurements to draw a scale map of the playground. Ask them to try to compute the area of the playground too.

- Send students to interview the school cooks. Have them ask how many gallons of soup it takes to serve everyone in the school. Then ask students to calculate the size (i.e., dimensions) of the pot(s) required to make the soup.

Connections to other subjects

Social Studies. Give students a map. Instruct them to calculate the distance (i.e., perimeter) of the Triangle Trade Route of the 1700s.

Social Studies/Geography. Have students practice calculating distances between cities. *(Hint: You may wish to make various assignments as though students will be traveling and need to make preparations.)* Provide maps and instruct students to calculate the distances in both metric and English (or British) units.

Alternative: You may wish to use a globe to measure the distances. You will need to create a scale. First, take a piece of string and measure the globe's circumference. Since the equator is approximately 40,000 kilometers, create a scale by dividing the length of the string into 40,000. The globe scale becomes this number of kilometers to 1 cm.

Home connections

Have the students draw layouts of the first floor of their houses, then measure the length and the width of the rooms in feet and calculate the floor area in square feet and square yards. *(Hint: You may wish to have students make a collage of the different floor plans.)*

Resources for teachers

Everyday Math by Marge Lindskog (Frank Shaffer Publications, 1996) See pp. 7, 17, and 19.
Area by J. Scivastava (Thomas Crowell, 1974)
Circles by M. Sitomer (Thomas Crowell, 1971)

1-cm Graph Paper

Stations Worksheet

Name ___________________________________

[Teacher note: List in the first column the figures/containers provided in the stations.]

Figure	**Area Station** *number of squares covered*	**Perimeter Station** *distance around the figure, in units*	**Volume Station** *number of cubes to fill the container*

Go the Distance!

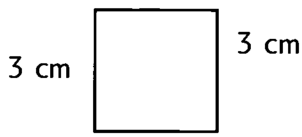
Name ______________________________

Note: Pi = 3.14

A. Compute the areas of the figures below. Remember that area is computed in square units.

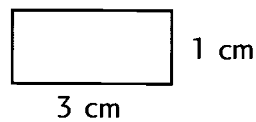
Area of square = __________

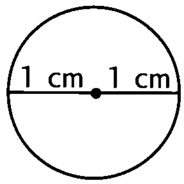
Formula: A = s^2



Area of rectangle = __________

Formula: A = L x W





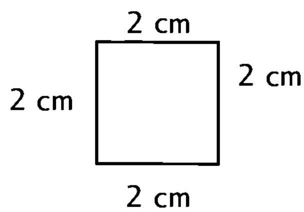
Area of circle = __________

Formula: A = Pi x r^2

B. Compute the perimeter of the figures below. Remember to keep track of the units!

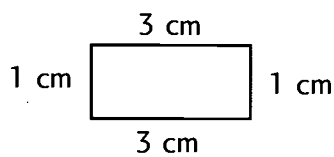
P = __________

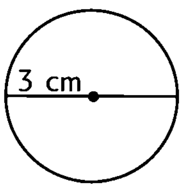
Formula: Perimeter is the sum of the length of all sides.



P = __________

Formula: Perimeter is the sum of the length of all sides.





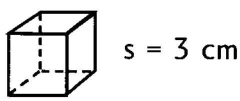
P = __________

Formula: Perimeter = Pi x d

C. Compute the volume of each figure. Remember that volume is measured in cubic units.

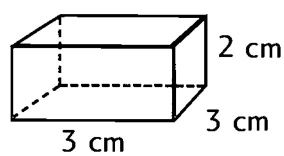
Volume = __________

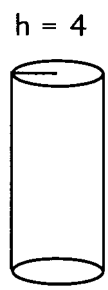
Formula: V = s^2



Volume = __________

Formula: V = L x W x H





Volume = __________

Formula: V = Pi x r^2 x h

Recording Chart

Name ______________________________

Hallway Measurements	In inches	Converted to feet	Converted to yards
Length			
Width			
Area	**In square inches**	**Converted to square feet**	**Converted to square yards**
Record calculations:			

Conversions:

12 inches = 1 foot
144 square inches = 1 square foot
9 square feet = 1 square yard

All Boxed In

About this learning activity

Students will find the surface area of common boxes (e.g., cereal, pasta, cake).

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II ○	IV ○	VI P	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Boxes (e.g., cereal, cake, pasta)
- Pencils and paper
- Markers
- Rulers (preferably metric)
- Scissors
- Tape (masking or transparent)
- Surface Area worksheets

Hint: You may wish to have students bring in boxes from home.

Engagement activity

Make sure each student has a box. Instruct students to examine their boxes and explain that they will be measuring the surface areas of their boxes (rectangular prisms). Ask students to study their boxes to find a way to keep them in 1 piece, but cut them so they lay flat. If no one discovers the answer, explain that they can cut **only on the seams** (i.e., where the boxes have been glued together). If time allows, check each student's cutting plan.

Exploration activity

Hint: This activity works best when completed individually.

1. Have the students cut their rectangular prisms (boxes) along the seam lines and lay them flat.

2. Ask them to examine their boxes and describe what they see.

3. Tell students to darken the seams of their boxes with markers, examine the boxes again, and describe what they see now. (Answer: Students should recognize that their boxes have become rectangles and/or squares.)

4. Help the students recognize that the surface area of a 3-dimensional figure (rectangular prism) is calculated by adding together the 2-dimensional figure areas.

5. Have the students use tape to reconstruct their boxes, then guide them in taking measurements (metric works best) with these instructions and questions:

   ? Find the front of your box, then measure its length and width.

   ? Calculate the area of your box front.

   ? If you know the area of the box front, what other area do you also know?

- If you find the area of the box top, what other area will you also know?
- If you find the area of the right side of the box what other side's area will you know?
- Once you know the area of every side, how can you find the surface area of your box?

6. Using your own box, let students help you determine its surface area. Be sure to complete a Surface Area worksheet on the board or overhead as you calculate your box's surface area.

7. Distribute the Surface Area worksheets and instruct students to find the surface area of their boxes.

8. When students have completed their worksheets, facilitate a class discussion with these and other questions:

    - When finding the surface of a rectangular prism, how many sides must we **calculate**?
    - How many sides must we calculate for a cube?

Real-life applications

Help students recognize how and why builders and remodeling professionals use surface area measurements to install appliances (e.g., ovens, dishwashers, refrigerators, freezers), bath tubs, showers, windows, and similar items.

Evaluation

Have the students determine the surface area of a given rectangular prism.

Journal assignment
Ask students to write instructions for finding the surface area of a rectangular prism, then try to do the same for a triangular prism.

Extension activity

- Let students practice finding the surface area of other objects (e.g., triangular prisms, cones, cylinders, spheres).

Home connections

Tell students that they are redecorating their bedrooms and they need to determine the amount of fabric needed for their new bedspreads. Note: This exercise offers practical learning about critical thinking because a few sides of the bed need not be covered by the spread and the spread must be long enough to touch the floor. You may wish to have students include calculations for their pillow cases for additional practice.

Surface Area

Name ____________________________

1. Locate your box front, then place the box in front of you with the front facing you. Show all of your calculations.

FRONT: Length = ________ Width = ________ Area of FRONT = ________

BACK: Length = ________ Width = ________ Area of BACK = ________

Calculation:

TOP: Length = ________ Width = ________ Area of TOP = ________

BOTTOM: Length = ________ Width = ________ Area of BOTTOM = ________

Calculation:

RIGHT SIDE: Length = ________ Width = ________ Area of RIGHT SIDE = ________

LEFT SIDE: Length = ________ Width = ________ Area of LEFT SIDE = ________

Calculation:

SURFACE AREA (of your rectangular prism):

SA = FRONT + BACK + TOP + BOTTOM + RIGHT SIDE + LEFT SIDE

Calculation:

All Finished? Don't forget units!

2. Now find the surface area of a fellow student's rectangular prism.

Who's prism are you using? ____________________

Front	Back	Top	Bottom	Right Side	Left Side
L =	L =	L=	L =	L =	L =
W =	W =	W =	W =	W =	W =
Surface Area					
SA = ______					

Who Wants Pi?

About this learning activity

Students will discover the value of pi by measuring the circumference of several objects.

Process skills

○ Building models
○ Categorizing or classifying
○ Communicating
○ Comparing
○ Controlling variables
○ Experimenting
○ Hypothesizing
○ Inferring
○ Interpreting data
○ Measuring
○ Observing
○ Ordering
○ Predicting
○ Reasoning
○ Recognizing relationships
○ Recording

Ohio mathematics proficiency strand(s)

I ○ III ○ V ○ VII ○
II ○ IV ○ VI P VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th-grade outcomes
○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th-grade outcomes
○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th-grade outcomes
○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Circular objects (e.g., cans, bottles, watch faces, clocks, cups, paper plates)
- String
- Ruler or meter stick
- Calculators
- Pencils and paper

Engagement activity

Tell students this joke: *What happens when you divide the circumference of a jack-o-lantern by its diameter?* (Answer: You get pumpkin "pi.") Explain to students that they will use objects with circles in them to learn about the relationship between the circumference of a circle and its diameter, and why pi is used in mathematical formulas.

Exploration activity

Hint: This activity works well both individually and with small groups.

1. Distribute the materials. Instruct the students to wrap string around the circular part of their objects and cut (i.e., the size of the circle).

2. Have students measure the strings they cut with a ruler or meter stick and record the findings.

3. Instruct them to measure the diameter of the circle with a ruler or meter stick and record the results.

4. Guide students through an example of making the calculations. Explain that circumference = pi x diameter and write *C = pi x d* on the board. Then, explain, "We want to find pi, so with a bit of mathematical magic we see that pi = C/d." Write *pi = C/d* on the board.

5. Using measurements you took from your own object or a student's measurements, write the formula again. Check your calculations with a calculator to ensure accuracy. Tell students that **they have just found pi**.

6. Instruct students to find pi for their objects and record their calculations.

7. Have students exchange objects several times and repeat the process.

8. After they have compiled a list of pi calculations, ask students how close their pi calculations came to 3.14. Then, facilitate a class discussion with these and other questions:

   - How many of your pi calculations were within .10 of 3.14?
   - Why do you think all of the pi calculations were not exactly 3.14? (Answer: See Explanation section.)
   - Why do you think we are taking the time to figure pi this way?
   - Why might it be important to manipulate formulas to identify other mathematical relationships?

Explanation

Students' solutions will vary depending on the object they are using. The accuracy of the pi number depends upon how carefully (i.e., accurately) they cut and measure their strings.

Evaluation

Observe how students wrap their strings around their objects and how they measure them.

Journal assignment
Ask students to explain how to find pi.

Extension activities

- Have students practice finding the **circumference** of circles.
- Have students practice finding the **area** of circles.
- Give students opportunities to practice finding the surface area of **spheres**.

Home connections

Have students show their parents that an apple pie is not the only pie found in their homes by calculating pi for such common household items as a lamp shade, round pillow, or round chair seat.

Box Builders

About this learning activity

Students will build boxes of specific dimensions, then measure their volumes.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II ○	IV ○	VI P	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- 2-cm grid paper (see p. 264)
- Tape
- Scissors
- 2-cm cubes (approximately 45 for each group)

Engagement activity

Organize students into pairs and distribute all materials **except the 2-cm cubes.** Instruct each team to use the grid paper to make an open box 4 squares long by 2 squares wide by 3 squares high. When teams finish building, ask them to list different ways they can measure the volume of their boxes. If no one suggests filling them with cubes and counting them, add this idea to the list.

Exploration activity

1. Keep students in the same teams used in the engagement activity. Distribute the 2-cm cubes.

2. Instruct teams to fill their boxes with the cubes, counting as they go. *(Hint: It takes 24 cubes to fill the box.)*

3. Ask the teams to explain how they filled their boxes. (Answer: Make a bottom layer of 8 cubes, then put 2 more layers on top of it.) Record the solution on the board in a table like the illustration.

Length	Width	Height	Volume	Cubes/layer	Layers
4	2	3	24	8	3
3	3	2	18	9	2
5	4	2	40	20	2

4. Instruct each team to build another open box 3 squares long by 3 squares wide by 2 squares high, then fill it with cubes, counting as they go. *(Hint: It takes 18 cubes to fill this box.)*

5. Ask the teams to explain how they filled their boxes. (Answer: Make a bottom layer using 9 cubes, then put a second layer on top of it.) Record this solution on the board too.

6. Have students repeat the process to build and measure a box 5 squares long by 4 squares wide by 2 squares high.

7. Instruct students to build rectangular solids of various sizes with their cubes and record their results in a table similar to the table you made on the board.

8. Help students identify the mathematical patterns in this exercise with these and other questions:

   - What pattern exists between the number of cubes per layer and the length and width of the boxes? (Answer: The number of cubes per layer = the length multiplied by the width.)

   - How can we write this as a mathematical formula? (Answer: A = L x W)

   - What are we calculating with this formula? (Answer: The area of the bottom or base of the boxes.)

   - What pattern exists between the volume and the cubes per layer and the number of layers? (Answer: Volume = cubes per layer times the number of layers)

   - How can we write this as a mathematical formula? (Answer: V = A x H or V = L x W x H)

Evaluation

Give each team or student a box with instructions to measure the area and volume.

Journal assignment
Ask students to write a journal entry that describes times when they might need to calculate volume in their daily lives.

Extension activity

- Let students practice finding volumes with other types of boxes like shoe boxes, cereal boxes, and gift boxes.

Connections to other subjects

Science. Discuss the relationship of volume and density with students, and let them experiment with volume and displacement.

2-cm Grid Paper

Measuring Around

About this learning activity

Students will freely explore the concepts of volume, time, and mass in learning stations.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II X	IV ○	VI P	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- 8-oz, 1-oz, pint, quart, and gallon containers (enough to supply the centers you create)
- Water, rice, sand, or similar materials (enough to supply the centers you create)
- Cleaning supplies (e.g., paper towels, broom, dust pan)
- Strips of bulletin board paper or wallpaper
- A good balance
- 1 set of metric masses (i.e., milligrams, grams, and a kilogram)

Engagement activity

Use the *Egyptian Measurement String*, which can be found at the following Internet address: http://www.forum.edu/`sarah/shapiro.knotted.string.html

Exploration activity

1. Create the stations described in the Explanation section in various areas around the room.

2. Have students follow the posted instructions to explore each station. Observe and rotate among the stations, providing guidance and encouragement.

3. When students have been given enough time to adequately explore the stations, facilitate a class discussion with these and other questions:

    ? What relationship exists between ounces and cups? Cups and pints? Pints and quarts? Quarts and gallons?

    ? How many cups would 1 gallon fill?

    ? How many ounces would 1 quart fill?

    ? What relationship exists between hours and days?

    ? What part of a day is 3 hours? Six hours? Eight hours? Twelve hours?

    ? How many milligrams make a gram?

    ? How many grams make a kilogram?

    ? What units are you multiplying or dividing by when you convert metric units?

Explanation

English Unit Station. Supply eight 1-oz containers, an 8-oz measuring cup, two 1-pint containers, four 1-quart containers, and a 1-gallon pitcher. Also provide water, rice, sand, or similar material. Post instructions that tell students to explore the relationships that exist. *(Hint: You may wish to create a Metric Unit Station with the same types of materials and containers.)*

Hint: The station activities are suggestions only. You may wish to adapt them to suit the needs of your students.

Time Chart Station. Supply a long strip of bulletin board paper or unpasted wallpaper (pattern side down). Post instructions that tell students to create a chart on the strip of paper that shows 60 minutes in an hour, 24 hours in a day, 7 days in a week, and how the weeks form a month. (A sample chart can be found on p. 269.)

Metric Mass Station. Supply the balance and masses. Post instructions that tell the students to experiment with the different masses and compare the results. *(Hint: Your high school science teacher can probably lend you the equipment needed or help you locate it.)*

Evaluation

Have the students create a *Handbook of Weights and Measures* that illustrates what they learned about various units of measure from working in the centers.

Journal assignment
Have students summarize what they learned about units of measure from the station activities.

Extension activity

- Tell students to visit various retail stores to study specific units of measure (e.g., lengths, weights, capacities). Have them report their findings concerning the various items studied, including the best unit or most common amount in which the product(s) studied is sold.

Connections to other subjects

Social Studies/Geography. Assign mapping activities that require students to convert units of measure.

Science. Conduct lab activities that involve changing units within a measurement system.

Home connections

Have students measure a room in their homes, then ask family members to help them locate pricing information for various floor coverings. Have them calculate in both yards and square feet the amounts of materials needed and the costs. Let students report their findings to the class.

Resources for teachers

Measurement by the Foot, Big Book of Everything, Fifth Grade by Mae Fuller (Instructional Fair Inc., 1995)
Sandwich Stuff, Everyday Math Grades 4–6 by Marge Lindskog (Frank Shaffer Publications, 1996)
How Much and How Many, the Story of Weights and Measures by J. Bendick (Franklin Watts, 1989)
Time and Clocks by H. Breiter (Raintree Publishers, 1978)

Sample Time Chart

Month of ____________________

Weeks
Each 24-hour period is $\frac{1}{7}$

Days $\frac{1}{24}$ $\frac{2}{24}$ ($\frac{1}{12}$)

Hours 1 2

Minutes 60 120

Measuring in Metric

About this learning activity

Students will take metric measurements of the classroom and objects inside the classroom.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II X	IV ○	VI P	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Rulers with cm markings
- Meter sticks (enough for student pairs or groups of 4)
- Paper and pencils
- Measuring Objects worksheets
- Assessment Sheets

Engagement activity

Discuss the metric system and its units of measure (e.g., mm, cm, m, km) with students. Discuss the appropriate units to use for measuring certain distances. For example, use km to measure the distance a car would drive from St. Louis to Columbus, m to measure a student's height, cm for the length of a student's book, and mm for the length of a fingernail.

Exploration activity

1. Organize students into pairs or groups of 4. Then, divide the teams into **3 categories**—category #1 teams will measure the room with a **ruler**, category #2 teams will measure the room with a **meter stick**, and category #3 teams will measure the room with a team member's foot (i.e., **paces**). List the 3 categories on the board.

Hint: You may wish to have teams count off by 3s to divide them among the categories.

2. Instruct teams to **measure** the room according to their assigned categories, recording the results.

3. After the students have measured the classroom, discuss which **unit of measure** was the best to use for this exercise.

4. Have students **convert** the cm measurement to meters and the foot paces to meters (i.e., measure the foot in cm and convert to meters).

5. Help students identify/review making conversions in the metric system (i.e., 10 mm = 1 cm, 1000 mm = 1 m, 100 cm = 1 m, 1000 m = 1 km). List the conversions on the board as students identify them.

6. Tell students that they will now measure a variety of objects in the room. Distribute the Measuring Objects worksheet and give students ample time to complete the exercise.

7. Let teams report their findings to the class and discuss the results. Help them discover and correct any errors.

8. Distribute the Assessment Sheets. Explain to students that converting from a smaller unit to a larger unit requires dividing, and converting from a larger unit to a smaller unit requires multiplying.

9. Compute the first 3 problems on the Assessment Sheet with the students. Then, let students complete the worksheet.

Real-life applications

Help students relate the learning in this activity to estimating money amounts with cents. Let them discuss how they might use estimating in this manner in their daily lives.

Evaluation

Grade students' Assessment Sheets.

Journal assignment
Have students answer some or all of these questions in their journals:

- How does the metric system differ from the traditional U.S. system?
- Which system is easier to use?
- Which system do you think the U.S. uses the most? Why?

Extension activities

- Help students discover some of the lesser-known metric system units like hectometer, decameter, and manometer.
- Provide activities that allow students to use the metric units for weight (kg, g, mg) and capacity (kL, L, mL).

Connections to other subjects

Science. Let students identify the appropriate unit(s) of measure for measuring space distances. (Answer: Light-years)

Social Studies. Have students complete research projects concerning customary measurement system origins and metric system origins. Let students debate which system should be used worldwide.

Measuring Objects

Name(s) ______________________________

Instructions

1. Measure objects with rulers and meter sticks, then combine the measurements of the appropriate number of objects to satisfy the requirements listed below.

2. Record each object measured and its measurement.

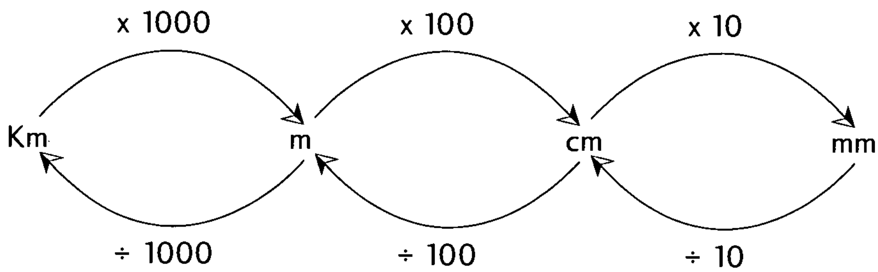
3. Record the total measurement of the combined objects.

4. Find:

   - 5 objects that total 1 m ______________________________
   - 3 objects that total 25 cm ______________________________
   - 7 objects that total 50 cm ______________________________
   - 2 objects that total 20 mm ______________________________
   - 6 objects that total 75 cm ______________________________
   - 4 objects that total 1000 mm ______________________________
   - 3 objects that total 1 m, 30 cm ______________________________

Assessment Sheet

Name ______________________________



1. 10 mm = ________ cm

2. 1 m = ________ mm

3. 200 cm = ________ m

4. 2465 m = ________ km

5. 6 km = ________ m

6. 100 cm = ________ mm

7. 2000 mm = ________ m

8. 3 m = ________ cm

9. 365 m = ________ mm

10. 3 km = ________ cm

11. 49,600 mm = ________ km

12. 300 m = ________ mm

Assessment Sheet
Answers

1. 1 cm
2. 1000 mm
3. 2 m
4. 2.465 km
5. 6000 m
6. 1000 mm
7. 2 m
8. 300 cm
9. 365,000 mm
10. 300,000 cm
11. 0.0496 km
12. 0.3 mm

"Anglers" and Their Protractors

About this learning activity

Students will find angles in their classroom and measure them with protractors.

Process skills

○ Building models
○ Categorizing or classifying
○ Communicating
○ Comparing
○ Controlling variables
○ Experimenting
○ Hypothesizing
○ Inferring
○ Interpreting data
○ Measuring
○ Observing
○ Ordering
○ Predicting
○ Reasoning
○ Recognizing relationships
○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II X	IV ○	VI P	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Analog alarm clock
- Overhead projector
- Blank transparency
- Erasable overhead markers
- Clear plastic protractor (The common student variety works well.)
- Paper and pencils
- Protractors (1 for each team of students)
- Angler Worksheets

Engagement activity

Display an analog alarm clock. Ask students to list the times of the day when the hands of an analog clock form a 90° angle, a 180° angle, a 45° angle, a 60° angle, and a 0° angle.

Exploration activity

1. Ask students to describe an angle. (Answer: An angle is 2 rays with a common end point.) If desired, allow some students to draw various kinds of angles on the board.

2. Explain that angles can be found everywhere. Help students make a list of angles found in the classroom.

3. Using an overhead projector, draw a line segment across a blank transparency to form a horizon. Place a point halfway across the line segment and label it, *A*.

4. Place the point of a transparent protractor on point A, demonstrating the 180° angle. Then, explain how to measure that angle with the protractor.

5. Draw another line segment perpendicular to the horizontal line and intersecting through point A. Put the point of the protractor on point A again to show students the 90° angle. Then, explain how you placed the protractor to measure the new angle.

6. Now, explain to students that sometimes we must extend the rays of an angle to measure it. Demonstrate this concept, then demonstrate that angles can open in any direction by drawing, extending, and measuring various angles.

7. Organize students into teams and distribute the Angler Worksheets. Instruct teams to find at least 15 angles measuring **other than** 90° or 180° in the classroom. Note: Students may need to extend the rays of some angles with common objects (e.g., rulers, meter sticks) to measure them. Also, don't stop groups from using the same objects (see Step 8).

8. Let teams review their object lists and measurements with the class. When students discover that 2 teams measured the same object, have them compare the measurements. If they are inconsistent, have teams remeasure to check their work. Note: If no agreement can be reached among the teams and/or class, list the object on the board and check it later for a final ruling.

Evaluation

Give a pencil-paper-protractor test.

Journal assignment
Ask the students to sketch a geometric toothpick design in their journals, then write descriptions of the various angles they employ.

Extension activity

☼ Invite a local carpenter (or similarly-talented parent) to demonstrate the use of angles with a miter box saw. Have him/her saw small molding strips at various angles to make several styles of picture frames. Let the students manipulate the pieces of wood to explore the angles produced by the various cuts of the saw.

Home connections

Ask parents to take a camera on a Sunday drive and let their students photograph interesting angles. Have students share the pictures with the class.

Resources for teachers

ASK Dr. Math (found on the world wide web)

Angler Worksheet

Name ______________________________

Object Examined	Angle Size

Notes

Fun with Math:
Real-Life Problem Solving for Grades 4–8

Strand VII: Estimation and Mental Computation

Learning Activities **Page**

Notes

Front-end Estimation

About this learning activity

Students will estimate purchases for a $2000 shopping spree, then practice front-end estimation with a variety of number and word problems.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I X	III X	V ○	VII P
II X	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Mail-order catalogs
- Paper and pencils
- Calculators

Engagement activity

You may wish to help students review the mathematical principles of estimation.

Exploration activity

1. Distribute the materials. Explain to students that they are going on a shopping spree and can spend $2000 on anything they want, but they must plan their purchases and stay within the $2000 allowance.

2. Instruct students to choose purchases from the catalogs, listing the name and price of each item on their order blanks (i.e., paper). Have them round prices to the hundreds place and estimate the total amount they will spend.

3. After students complete their shopping sprees, ask volunteers to share what they "bought" and explain the strategies they used to stay within the $2000 budget. If no one cites **front-end estimation** as a strategy, explain the concept and how it could be/was used for the shopping spree.

4. Go around the room, letting all students share their total estimates. Give special recognition to those who come closest to spending the allowance **without going over**.

5. Next, help students review/practice estimating sums and differences by **rounding** to the nearest thousands place with the following problems: *(Hint: List them on the board or overhead.)*

   - 3423 + 2742 (Answer: 6000)
   - 5692 + 6221 (Answer: 12,000)
   - 9294 – 3401 (Answer: 6000)
   - 7735 – 2343 (Answer: 6000)

6. Have students **estimate** more sums and differences to the thousands place, then calculate the **exact** answers. Provide problems such as:

   - 4519 + 6281 (Estimate: 11,000; Actual: 10,800)
   - 7691 – 5346 (Estimate: 3000; Actual: 2345)

7. Facilitate a discussion about estimating with these and other questions:

   - ? How do the estimates compare to the exact answers?
   - ? How could we calculate estimates that are closer to the actual totals? (Answer: Round to the hundreds or tens place.)

Real-life applications

Present the following problem on the chalkboard or overhead:

> *Mary plans to work with her neighborhood theater group to present a classic play. The group's budget is $4500. Can the group afford to purchase scenery materials at $2688, tools at $419, costumes at $312, and props at $539?*

Have students use front-end estimation to find the answer.

Evaluation

Have students create front-end estimation problems for their classmates to solve. Assess both the problems and solutions.

Extension activity

- Provide supermarket advertisements and a dinner menu. Have students work in pairs to plan a dinner meal, considering cost, taste, and nutrition. Then, instruct **each student** (not team) to make a grocery list for the team's dinner, including estimated costs for each food item. Have partners exchange lists and estimate the total grocery bills. Finally, tell partners to find the food items on their lists (as many as possible) in the supermarket advertisements and estimate the actual costs of their dinners.

Journal assignment

Have students explain how to use front-end estimation to estimate 1704 + 4198 + 2003. Have them describe a scenario in which a total of these numbers would need to be estimated. (For example, a company manufactured refrigerators over 3 years; 1704 in the first year, 4198 in the second, and 2003 in the third. Approximately how many were manufactured in those 3 years?)

Home connections

Have each student plan a family car trip and plot it on a map, showing at least 4 destinations. Instruct each student to estimate the mileage between the destinations, then the total miles the family will travel.

Resources for teachers

About Teaching Mathematics, A K–8 Resource by Marilyn Burns (Math Solutions Publications)

30 Wild and Wonderful Math Stories to Develop Problem-Solving Skills by Greenberg (Scholastic Inc., 1992)

Math Games & Activities, Volume 2, Toolbox of Duplicating Design for Middle and Upper Grade Elementary School Teachers by Paul Shoecraft (Dale Seymour Publications, 1984)

Hands-on Math! Ready-to-Use Games & Activities for Grades 4–8 by Frances M. Thompson (The Center for Applied Research in Education, 1994)

Estimating Areas and Volumes

About this learning activity

Students will first estimate the surface area and volume of several objects, then measure and calculate the actual surface areas and volumes.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II ○	IV ○	VI P	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Several objects of different shapes and sizes (Samples are provided on the Estimation worksheet, p. 290.)
- Estimation worksheets (or plain paper)
- Measuring devices (e.g., rulers, meter sticks)
- Calculators

Engagement activity

You may wish to help students review the mathematical principles and formulas involved in finding volume and surface area.

Exploration activity

Hint: Sample objects are provided on the Estimation worksheet on p. 290. Answers can be found on p. 291.

1. Distribute a few objects to each student.

2. Instruct students to estimate the length, width, and/or height of each object, recording their estimates on their worksheets (or papers).

3. Then, have students estimate the surface area and volume of the objects. Explain that they may use the formulas given, but may not use calculators. Again, tell them to record their estimates.

4. Distribute the measuring devices and have students measure and record the actual length, width, and/or height of each object.

5. Distribute the calculators and have the students calculate and record the surface areas and/or volumes of their objects.

6. Tell students to compare their estimates against their measurements and calculations, marking whether the estimates were greater or less than the actual findings.

7. Give students an opportunity to share their results. Then, ask them to name real-life situations that might require estimating surface areas or volumes.

Real-life applications

Help students identify these estimation situations:

- When shopping with limited cash, you need to estimate your bill to ensure that you have enough money to pay for your purchases.
- When painting your house, you need to estimate its surface area to calculate the proper amount of paint to buy.

Evaluation

Have each student bring an object from home that can be measured in whole numbers, with the measurements taped face down to the bottom of the object. Instruct students to leave their objects on their desks, and move from desk to desk to estimate and record the dimensions, volume, and surface area of each object. When estimates are finished, have the students return to their desks and read aloud the measurements from the bottoms of their objects. Have students exchange papers to compare estimates and actuals (or check the student papers yourself). *(Hint: You may wish to do this exercise in the school gym or cafeteria, using tables or the floor.)*

Journal assignment
Instruct students to write helpful hints for estimating the height, length, or width of an object for other students to use.

Extension activity

- Instruct students to work in pairs to check the truth of this theory: *A person's height is the same as the length of his/her extended arms (fingertip to fingertip).* Have students estimate the 2 measurements for their partners, then measure each other. Check how many students proved and disproved the theory.

Home connections

Have the students estimate the surface areas of their bedroom walls, then the window and door areas. Instruct students to ask family members to help them take the actual measurements and compare them to their estimates.

Resources for teachers

Students will be using pi in their calculations. For more information on pi, see http://www.cecm.sfu.ca/pi/.

Estimation Worksheet

Name ______________________________

Instructions

First, estimate the values for the measurements below and compute the area or volume. Next, measure the length, width, height, depth or diameter of the given object and calculate the area or volume. Compare your results. How close was your estimate?

	Estimated width	Estimated length	Estimated area	Actual width	Actual length	Actual area
Area of 1 side of a ½-gallon milk carton						
Area of a large picture frame						
Area of a small picture frame						
Area of a television screen						
Surface area of 1 side of a brown paper grocery bag						

	Estimated width	Estimated length	Estimated area	Actual width	Actual length	Actual area	Volume
Volume of a brown paper grocery bag							
Volume of a coffee cup (use 3 as an approximation for pi)							

Estimation Worksheet Answers

	Estimated width	Estimated length	Estimated area	Actual width	Actual length	Actual area
Area of 1 side of a 1/2-gallon milk carton				4 in	8 in	32 sq in
Area of a large picture frame				9 in	12 in	108 sq in
Area of a small picture frame				3 in	4 in	12 sq in
Area of a television screen*				16 in	12 in	192 sq in
Surface area of 1 side of a brown paper grocery bag				12 in	17 in	204 sq in

	Estimated width	Estimated length	Estimated area	Actual width	Actual length	Actual area	Volume
Volume of a brown paper grocery bag				12 in (7 in depth)	17 in	204 sq in	1428 cu in
Volume of a coffee cup (use 3 as an approximation for pi)				3 in (diameter)	4 in		27 cu in

*TV screens are always designed with a ratio of 4 to 3.

Rounding Decimals

About this learning activity

Students will round decimals to estimate the amount of cargo that can be shipped in a moving van, then practice rounding decimals with number and word problems.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII P
II X	IV ○	VI ○	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Cargo Lists
- Calculators
- Paper and pencils
- Construction paper
- Estimate Decimal Sums and Differences worksheets

Engagement activity

1. Distribute a Cargo List (p. 296) to each student (or display it on an overhead projector). Explain to the students that all of this cargo must be moved to a new home in an 800-lb-capacity truck.

2. Have students work in teams and **use estimating** to list the cargo items by truckload, seeking the fewest number of trips possible. Tell students they may not use calculators.

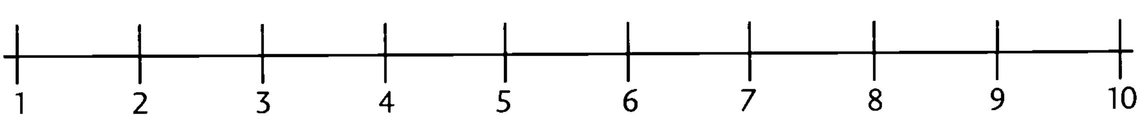
3. When all lists are complete, instruct the teams to exchange lists and use the calculators to check them.

4. Facilitate a class discussion that lets students share their methods for deciding how to organize the cargo. Help them identify the most successful estimation strategies, based on the accuracy of their lists.

> Alternative: You may wish to make Steps 1 and 2 of the exploration activity serve as the engagement activity, and let students complete the Cargo List exercise as part of the exploration activity.

Exploration activity

1. Distribute the construction paper. Have students draw a number line with end points of 1 and 10, and increments of 10, on their papers (see illustration).



2. On the board or overhead projector write the addition problem 2.6 + 1.3 and demonstrate how to estimate the sum using the number line. [Answer: Start at 2.6, go to the nearest whole number (3), round the addend (1.3) to the nearest whole number (1), then use the number line to add the 2 rounded numbers.]

3. Give students a few sample problems to solve with their number lines. Observe to ensure each student grasps the concepts.

4. Present this word problem to students:

   Brenda wants to buy a notebook for $3.98, a highlighter for $.69, a package of pencils for $1.39, and a box of markers for $2.79. She has $10. Estimate whether she has enough cash to purchase all the school supplies.

5. Then, tell students that Brenda wants to estimate the cost difference between the box of markers and a box of crayons that costs $1.49. Ask them what strategy Brenda could use to estimate an answer for this subtraction problem.

6. Facilitate a class discussion about rounding decimals when estimating answers with these and other questions:

   ? How do you decide which place to round to when you estimate a decimal sum or difference?

   ? Does the level of accuracy required for the estimation matter? (Answer: Yes, especially in problems like Brenda's. If the estimate was off by less than $1, she could suffer embarrassment at the cash register.)

7. Distribute and ask students to complete the Estimate Decimal Sums and Differences worksheet. Note: If you want to use this worksheet for evaluation purposes, have students complete it individually.

Evaluation

Grade students' Estimate Decimal Sums and Differences worksheets.

Journal assignment
Have students explain the rules for estimating decimal sums and differences in their journals.

Extension activity

☼ List each student's age in years and months. Then, have students use estimation to find the combined number of years the class has lived.

Connections to other subjects

Science. Use scientific data to make estimations.

Home connections

Have family members help students write the weekly grocery list and provide a food budget. Instruct students to use the grocery section of the newspaper to estimate the total cost of the foods on their lists and compare the totals against their allocated budgets. If any estimates exceed budgets, have those students trim their grocery lists. Finally, have students use a calculator to calculate their actual grocery bill totals and compare them to their estimates.

Resources for teachers

About Teaching Mathematics, A K–8 Resource by Marilyn Burns (Math Solutions Publications)

30 Wild and Wonderful Math Stories to Develop Problem-Solving Skills by Greenberg (Scholastic Inc., 1992)

Math Games & Activities, Volume 2, Toolbox of Duplicating Design for Middle and Upper Grade Elementary School Teachers by Paul Shoecraft (Dale Seymour Publications, 1984)

Hands-on Math! Ready-to-Use Games & Activities for Grades 4–8 by Frances M. Thompson (The Center for Applied Research in Education, 1994)

Cargo List

Furniture	Number	Total Average Weight
Bed	3	325.8
Dresser	2	115.25
Chest of drawers	3	80.65
Couch	3	246.50
Easy chair	4	75.3
Hutch	1	163.8
Dining room table	1	285.5
Dining room chair	6	15.3
Kitchen table	1	110.25
Kitchen chair	4	12.5
Coffee table	2	21.25
End table	5	26.2
Refrigerator	1	478.75
Stove	1	277
Washer	1	254.85
Dryer	1	145.4

Estimate Decimal Sums and Differences

Name ______________________________

1. $\begin{array}{r} 3.456 \\ +\ 4.603 \\ \hline \end{array}$	2. $\begin{array}{r} 8.532 \\ -\ 5.293 \\ \hline \end{array}$	3. $\begin{array}{r} 4.5490 \\ +\ 2.5958 \\ \hline \end{array}$
4. $\begin{array}{r} 8.304 \\ -\ 7.294 \\ \hline \end{array}$	5. $\begin{array}{r} 6.406 \\ +\ 3.596 \\ \hline \end{array}$	6. $\begin{array}{r} 3.394 \\ +\ 4.593 \\ \hline \end{array}$
7. $\begin{array}{r} 9.392 \\ -\ 7.394 \\ \hline \end{array}$	8. $\begin{array}{r} 4.394 \\ -\ 1.493 \\ \hline \end{array}$	9. $\begin{array}{r} 8.293 \\ +\ 5.398 \\ \hline \end{array}$
10. $\begin{array}{r} 4.209 \\ -\ 3.987 \\ \hline \end{array}$	11. $\begin{array}{r} 7.938 \\ +\ 8.049 \\ \hline \end{array}$	12. $\begin{array}{r} 6.392 \\ -\ 4.234 \\ \hline \end{array}$
13. $\begin{array}{r} 3.395 \\ +\ 9.943 \\ \hline \end{array}$	14. $\begin{array}{r} 9.234 \\ -\ 5.398 \\ \hline \end{array}$	15. $\begin{array}{r} 8.392 \\ +\ 2.234 \\ \hline \end{array}$
16. $\begin{array}{r} 4.293 \\ +\ 9.987 \\ \hline \end{array}$	17. $\begin{array}{r} 4.398 \\ -\ 3.332 \\ \hline \end{array}$	18. $\begin{array}{r} 5.439 \\ +\ 6.544 \\ \hline \end{array}$

19. James bought a hamburger for $2.39, small fries for $.99, and a large cola for $.75. About how much did he spend for lunch?

20. Jennifer went shopping at the mall and purchased the following items: a pair of jeans for $45, a plaid shirt for $29, and a pair of hiking boots for $64. Estimate the total cost of Jennifer's purchases and the amount of change she received from the $200 she gave the cashier.

Estimate Decimal Sums and Differences
Answers

1. 3.456 + 4.603 = 8

2. 8.532 − 5.293 = 4

3. 4.5490 + 2.5958 = 8

4. 8.304 − 7.294 = 1

5. 6.406 + 3.596 = 10

6. 3.394 + 4.593 = 8

7. 9.392 − 7.394 = 2

8. 4.394 + 9.493 = 13

9. 8.293 − 5.398 = 3

10. 4.209 + 3.987 = 8

11. 7.938 + 8.049 = 16

12. 6.392 − 4.234 = 2

13. 3.395 + 9.943 = 13

14. 9.234 − 5.398 = 4

15. 8.392 + 2.234 = 10

16. 4.293 + 9.987 = 14

17. 4.398 − 3.332 = 1

18. 5.439 + 6.544 = 12

19. James bought a hamburger for $2.39, small fries for $.99, and a large cola for $.75. About how much did he spend for lunch?

    $4

20. Jennifer went shopping at the mall and purchased the following items: a pair of jeans for $45, a plaid shirt for $29, and a pair of hiking boots for $64. Estimate the total cost of Jennifer's purchases and the amount of change she received from the $200 she gave the cashier.

    $140; $60

Estimating Fractions and Decimals

About this learning activity

Students will measure lines and estimate the sums and differences between line sets, then use store advertisements to estimate sums and differences between products.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII P
II ○	IV ○	VI X	VIII ○

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Advertisements with prices that include cents (e.g., department store, grocery store) (1–2 per student)
- Rulers
- Estimating Fractions worksheets
- Estimating Decimals worksheets
- 1 box of cookies *(Hint: Bite-sized cookies will **not** work with this exercise.)*

Engagement activity

1. Ask a student to come to the front of the room. Give your volunteer a cookie and instruct him/her to take 1 bite. Have the volunteer walk around the room, displaying the remains of the cookie. Instruct the class to estimate how much of the cookie remains from these choices: **almost all, half, or almost none.**

2. Repeat this activity with a few more volunteers. Then, ask students to **quantify** the choices (i.e., what number do you equate with an **estimate** for each choice?) (Answers: Almost all = 1; half = $1/2$; and almost none = 0)

3. Tell students to remember these concepts because they will use them to estimate fractions and decimals.

Exploration activity

1. Distribute the Estimating Fractions worksheets. Explain to students that they will be **measuring the lines** on their worksheets in inches, and will be estimating the sum, difference, and product of the line combinations **to the nearest $1/2$ or whole number** (i.e., if the number is below $1/4$, they will round down; if the number is $1/4$ to below $3/4$, they will round to the nearest $1/2$; and if the number is $3/4$ and above, they will round to the next greatest whole number). Note: Review the terms *sum* (addition), *difference* (subtraction), *product* (multiplication), and *quotient* (division) if necessary.

2. Have students **measure the sample line** and ask them to report the results. (Answer: $3\,1/4$ in)

3. Ask students what **length** they must **estimate** for the sample line, based on this activity's parameters (i.e., measure to the nearest $1/2$ or whole number). (Answer: $3\,1/2$, because the number is $1/4$ to below $3/4$)

4. Tell students to look at line combinations A-F. Explain that they will estimate the **sum of those lines** (i.e., add A + B, C + D, E + F) and they want to find the **most accurate** estimates.

5. Facilitate a discussion that helps students conclude that they can round each line individually, then add, but that would not result in the most accurate estimate. Use an example like this: a line might measure $3^3/_4$ in and be rounded to 4 in, and a second line might measure $3^1/_4$ in and be rounded to $3^1/_2$ in, making the estimated sum $7^1/_2$ in. The actual sum is 7 in.

6. Help students recognize how to **change their estimation strategy** to get closer to the actual sum by encouraging them to keep in mind the amount they are adding to an estimated number. Use the same example: for $3^1/_4$, students add $^1/_4$ to create an estimate of $3^1/_2$. They need to hold the $^1/_4$ amount in their minds and look for an opportunity to reduce this amount when they estimate the second number (i.e., $3^3/_4$ in is **rounded down** to $3^1/_2$). Note: Students are likely to struggle with this concept, so help them practice it several times and apply it to real-life situations that will help them assimilate it. *(Hint: Problem 7, line combination K–L, uses this concept.)*

7. Let students review the rest of the worksheet and ask questions. Confirm that they understand that they must subtract (estimate **differences** for) line combinations G–L and multiply (estimate **products** for) line combinations N–R. Then, instruct students to complete the worksheet individually or in pairs.

8. As students finish the Estimating Fractions worksheet, distribute 2–3 advertisements to each student and explain that the class will now estimate decimals. Let students look through their advertisements.

9. Distribute the Estimating Decimals worksheets and explain to students that they will use their advertisements to estimate sums, differences, products, and quotients. Then, instruct students to record product names and prices; their estimates for each product price; and their estimates of the sum, difference, product, or quotient on their Estimating Decimals worksheets. (Sample problem: Students must estimate a sum of $20 using 3 items. They select a bath towel for $4.44, a compact disc for $11.00, and laundry detergent for $5.00. These items allow them to estimate a sum of $20 and calculate an actual sum of $20.82.)

10. When estimates are complete, ask students to **calculate the actual prices and totals**, then compare these to their estimates.

11. See if anyone can recognize that the **same rules apply** for decimals that applied for fractions. [Answer: If the number was below 0.25 ($\frac{1}{4}$), they rounded down; if the number was 0.25 ($\frac{1}{4}$) to below 0.75 ($\frac{3}{4}$), they rounded to 0.50 ($\frac{1}{2}$); and if the number was 0.75 ($\frac{3}{4}$) or above they rounded up to the next whole number.]

12. Help students check their worksheets and discuss their findings.

Explanation

Estimating Fractions Answers

	Actual	Estimation		Actual	Estimation	
Line A	$1\frac{3}{4}$ in	2 in	Line B	$\frac{5}{8}$ in	$\frac{1}{2}$ in	Sum: $2\frac{1}{2}$ in
Line C	$2\frac{1}{4}$ in	$2\frac{1}{2}$ in	Line D	$\frac{15}{16}$ in	1 in	Sum: $3\frac{1}{2}$ in
Line E	$3\frac{1}{16}$ in	3 in	Line F	$1\frac{7}{8}$ in	2 in	Sum: 5 in
Line G	$2\frac{10}{16}$ in	$2\frac{1}{2}$ in	Line H	$1\frac{1}{8}$ in	1 in	Difference: $1\frac{1}{2}$ in
Line I	$2\frac{1}{16}$ in	2 in	Line J	$\frac{5}{16}$ in	$\frac{1}{2}$ in	Difference: $1\frac{1}{2}$ in
Line K	$1\frac{3}{16}$ in	1 in	Line L	$\frac{3}{4}$ in	1 in	Difference: $\frac{1}{4}$ in
Line M	$1\frac{11}{16}$ in	$1\frac{1}{2}$ in	Line N	$2\frac{3}{16}$ in	2 in	Product: 3 in
Line O	$3\frac{3}{16}$ in	3 in	Line P	$1\frac{15}{16}$ in	2 in	Product: 6 in
Line Q	$2\frac{7}{16}$ in	$2\frac{1}{2}$ in	Line R	$1\frac{10}{16}$ in	$1\frac{1}{2}$ in	Product: 4 in

The answers for the Estimating Decimals worksheet will differ from student to student because the data varies (by the advertisements).

Real-life applications

Students use estimating skills daily without realizing it. Remind students about situations in which they may have estimated how much pizza or pie was left, whether they had enough money to buy some candy, or whether they had enough time to visit a friend before dinner. Let students provide additional examples.

Evaluation

Review students' Estimating Fractions and Estimating Decimals worksheets.

Journal assignment
Have students describe how they must balance additions and subtractions to numbers when estimating. (See the concept described in Step 6.) Encourage them to use examples to make this task easier.

Extension activity

- Tell students to imagine they are working as professional carpet installers and their current job is carpeting a floor that measures $18\frac{5}{8}$ ft by $20\frac{1}{4}$ ft. Carpet must be purchased by the whole or half foot. Explain that they have a very tough boss, who has a reputation for firing installers when they buy too much carpet. Instruct students to estimate the amount of carpet they should buy and explain their answers. (Answer: The area measures $377\frac{5}{32}$ sq ft. Standard estimation practices will result in rounding down, resulting in an order of 370 sq ft. However, in this case, students must round **up** to $377\frac{1}{2}$ sq ft.) See if students can explain why they must round up. Explain how important it is to have enough materials when completing such jobs.

Connections to other subjects

Economics. Have students learn how to create and maintain a budget by estimating their expenses for the month and calculating whether or not their income (e.g., allowance, odd jobs) will cover their expenses. You may wish to have students illustrate the results in a graph.

Home connections

Ask family members to let students practice estimation skills during trips to department and grocery stores.

Resources for teachers

Math in the Real World of Business and Living by Shirley Cook (Incentive Publications, 1996)

Estimating Fractions Worksheet

Name ______________________________

Note: Measure in inches.

Sample line: ______________________________
Estimation for sample line: __________

1. Estimate A + B Estimated Sum: __________

   A) ____________________ B) ______

2. Estimate C + D Estimated Sum: __________

   C) __________________________ D) __________

3. Estimate E + F Estimated Sum: __________

   E) ____________________________________ F) ____________________

4. Estimate G – H Estimated Difference: __________

   G) ______________________________ H) ____________

5. Estimate I – J Estimated Difference: __________

   I) _______________________ J) ____

6. Estimate K – L Estimated Difference: __________

   K) ______________ L) ________

7. Estimate M x N Estimated Product: ________

   M) ___________________ N) ________________________

8. Estimate O x P Estimated Product: ________

   O) ______________________________________ P) ____________________

9. Estimate Q x R Estimated Product: ________

   Q) ____________________________ R) __________________

Estimating Decimals Worksheet

Name ______________________________

		Product Name	Product Price	Estimation of Price
#1	Three items that total $15	a.		
		b.		
		c.		
			Estimated Sum: __________	
			Actual Sum: __________	
#2	Five items that total $50	a.		
		b.		
		c.		
		d.		
		e.		
			Estimated Sum: __________	
			Actual Sum: __________	
#3	Two items that total $5	a.		
		b.		
			Estimated Sum: __________	
			Actual Sum: __________	
#4	Six items that total $30	a.		
		b.		
		c.		
		d.		
		e.		
		f.		
			Estimated Sum: __________	
			Actual Sum: __________	

		Product Name	Product Price	Estimation of Price
#5	Seven items that total $100	a.		
		b.		
		c.		
		d.		
		e.		
		f.		
		g.		
			Estimated Sum: ______________ Actual Sum: __________________	
#6	Two items with a difference of $6	a.		
		b.		
			Estimated Difference: ______________ Actual Difference: ________________	
#7	Two items with a difference of $20	a.		
		b.		
			Estimated Difference: ______________ Actual Difference: ________________	
#8	Two items with a difference of $10	a.		
		b.		
			Estimated Difference: ______________ Actual Difference: ________________	
#9	Two items with a difference of $50	a.		
		b.		
			Estimated Difference: ______________ Actual Difference: ________________	

		Product Name	Product Price	Estimation of Price
#10	Three items with a product of $15	a.		
		b.		
		c.		
			Estimated Product: ________	
			Actual Product: ________	
#11	Four items with a product of $75	a.		
		b.		
		c.		
		d.		
			Estimated Product: ________	
			Actual Product: ________	
#12	Five items with a product of $150	a.		
		b.		
		c.		
		d.		
		e.		
			Estimated Product: ________	
			Actual Product: ________	
#13	Two items with a product of $20	a.		
		b.		
			Estimated Product: ________	
			Actual Product: ________	

		Product Name	Product Price	Estimation of Price
#14	Two items with a quotient of $5	a.		
		b.		
			Estimated Quotient: ______ Actual Quotient: ______	
#15	Two items with a quotient of $10	a.		
		b.		
			Estimated Quotient: ______ Actual Quotient: ______	
#16	Two items with a quotient of $25	a.		
		b.		
			Estimated Quotient: ______ Actual Quotient: ______	
#17	Two items with a quotient of $2	a.		
		b.		
			Estimated Quotient: ______ Actual Quotient: ______	

Strand VIII: Data Analysis and Probability

Learning Activities **Page**

Notes

What Are the Chances?

About this learning activity

Students will count and sort a multicolored item to predict outcomes, then check the accuracy of their predictions.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○ III ○ V ○ VII ○
II ○ IV ○ VI ○ VIII P

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- M&Ms-type candies (or Skittles, jelly beans, buttons, colored bingo chips, various types of beans)
- Plastic sandwich bags (1 per student)
- Probability Charts (1 per student)

Engagement activity

Write, "What are the chances?" on the board. Then, ask students:

? What are the chances that school will be dismissed at noon today? At the regular time?

? What are the chances that school will be closed on Wednesday? On Saturday and Sunday?

? What is probability? (Answer: Probability lets us predict and explain the likelihood or chances that something will or will not occur.)

Exploration activity

1. Put a handful of candies in each plastic bag. Distribute the bags and charts to the students and remind them not to eat the experiment!

2. Have students count the candies and record the totals on their charts.

3. Instruct students to sort and count the candies by color, record their findings in the second column of their charts, and return the candies to their bags.

4. As students finish, explain that you are about to ask them to select just 1 candy without looking. Ask them, "What is the probability that it will be yellow?" Note: Students will make different predictions based on the contents of their individual bags.

5. Ask students to explain how they made their predictions. List their ideas on the board. Some students may use a probability method (e.g., guessed based on the data in their charts). Be sure that all students hear and understand these methods as they are explained.

6. Discuss the probability column. Help students recognize that they can improve the accuracy of their predictions by using a formula to determine the color probability.

7. Facilitate a discussion that guides students to conclude that probability equals the number of successful ways divided by the total number—in this case, the number of colors divided by the total number of candies.

8. Ask a student volunteer to write a mathematical formula to represent this concept. (Answer: $P = \frac{C}{T}$, where P = probability, C = total colors, and T = total candies.)

9. Have students use this formula to predict the probability that they will choose each color from their bags. Tell them to record the results in the probability column of their charts.

10. Have students share their results with 1–2 other students. Then, help them discuss why their individual probability predictions differed. Let them eat their candies during the discussion.

Real-life applications

Have students use probability to make predictions about selecting socks from a drawer or sweaters from a closet, getting tickets to a popular event, and similar situations. Help them identify ways that adults use probability. Examples: Buying stocks and choosing cars or appliances (i.e., likelihood of breakdown vs. cost).

Evaluation

Check students' Probability Charts.

Journal assignment
Name an event and ask students to describe in their journals the process they would use to calculate the probability that the event will occur.

Extension activity

- Ask students to write probability problems for each of their other classes/subjects. Example: There are 25 students in Social Studies class and the teacher always directs questions to 3 students (different students each day). What is the probability that you will get called on today?

Connections to other subjects

Science. Help students study probability in relation to genetics.

Language Arts. Provide an exercise in which students can identify the probability of finding the letter "e," vowels, or certain consonants in words.

Home connections

Have students work with a family member to calculate the probability that certain events will happen at home. Example: What is the probability that the telephone will ring during dinner?

Resources for teachers

Probability Activities by Robert Lovell (Key Curriculum Press)
Mastermind Riddle Series, Fractions, Ratios, Proportions & Standard Measurement by Brenda Opie, Lory Jackson, and Douglas McAnin (Incentive Publications)
When Are We Ever Gonna Have to Use This? by Hal Saunders (Dale Seymour Publications)
Mathercise by Michael Serra (Key Curriculum Press, 1992)

Computer programs:

- *Everyday Math*
- *Super Solvers Outnumbered!* (The Learning Company)
- *Math Blaster Mystery* (Davidson)

Probability Chart

Name ______________________________

Total number of candies __________

	Quantity	Probability
Yellow		
Red		
Green		
Blue		
Purple		
Orange		
Brown		

Do You Have Any Hearts?

About this learning activity

Students will draw cards from a standard playing deck, record the occurrence of hearts, and predict the probability of hearts occurrences.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II ○	IV ○	VI ○	VIII P

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Standard decks of 52 playing cards (1 for each student)
- Paper and pencils
- Hearts Tally Sheets

Engagement activity

Working in teams or individually, have students shuffle their cards thoroughly, then deal 4 from the top, turning them face up in a row. Tally the number of hearts that were turned up. Then, help students total the possibilities (i.e., 4 multiplied by the number of decks used). Help students compare and discuss the 2 totals.

Exploration activity

Hint: This activity works best when completed individually.

1. Distribute the Hearts Tally Sheets and instruct students to complete it. Review the instructions with students, if necessary. Note: A sample tally chart can be found in the Explanation section.

2. As students complete the tally sheets, challenge them to predict the probability of dealing at least 1 heart in a 4-card deal. Let them use calculators and explain that they must show their work. *(Hint: You may wish to let students work in pairs or small groups during this portion of the activity.)*

3. Facilitate a class discussion with these and other questions:

    ? What is the probability of dealing at least 1 heart in a 4-card deal? (Answer: 0.6962 or approximately 70%. Thus, in 25 deals, 1, 2, 3, or 4 hearts should appear around 17 or 18 times. The probability of 0 cards being hearts is approximately 30%.)

    ? How did you compute the probability?

    ? What does your chart show?

    ? Would the results be the same if you were counting clubs instead of hearts? Explain.

Explanation

By collecting and recording the hearts data on the bar graph, students will see how such information can be used to predict probability.

Sample Hearts Tally Chart

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
X																								
X					X																			
X	X	X			X																			

Evaluation

Have students complete a similar card activity, instructing them to find the probability of dealing at least 1 royalty card (i.e., king, queen, jack). Note: A worksheet for this evaluation activity can be found on p. 320.

Journal assignment
Instruct students to discuss whether they think about the probability of outcomes in their daily lives, including why or why not.

Extension activities

- Remove all black cards from the playing decks and have students repeat the exploration activity.
- Repeat the exploration activity with pinochle or euchre decks.

Home connections

Have students work with family members to monitor the family mail. Have them record the total pieces of mail and the number of junk mail pieces delivered daily, then predict the probable amount of junk mail their families can expect to receive in future weeks.

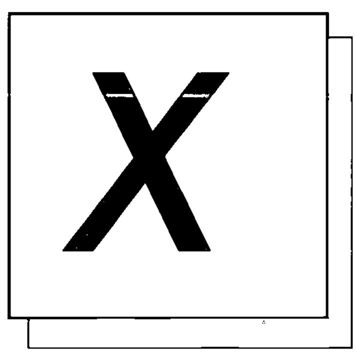
Hearts Tally Sheet

Name ______________________________

Instructions

1. Shuffle your deck of playing cards.
2. Deal 4 cards, turning them face up in a row.
3. Count the number of hearts that occur and put an "X" in the appropriate box in the chart below.
4. Repeat Steps 2 and 3 until you have completed 25 rounds. Shuffle the cards **only when you reach the end of the deck.**
5. Color or shade each box that holds an "X" to create a bar graph.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25



Royalty Worksheet

Name ______________________________

Instructions

1. Shuffle your deck of playing cards.

2. Deal 4 cards, turning them face up in a row.

3. Count the number of royalty cards (i.e., king, queen, jack) that occur and put an "X" in the appropriate box in the chart below.

4. Repeat Steps 2 and 3 until you have completed 25 rounds. Shuffle the cards **only when you reach the end of the deck.**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

5. Color or shade each box that holds an "X" to create a bar graph.

6. Using the data you collected and the bar graph you created, predict the probability of finding 1 royalty card in a 4-card deal. You may use a calculator, but you must show your work.

   Probability = __________

7. Is the probability of finding royalty different than the probability of finding hearts? Why or why not?

Card Predictors

About this learning activity

Students will predict probabilities for a variety of card-drawing situations.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II ○	IV ○	VI ○	VIII P

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Standard decks of 52 playing cards (1 for each student)
- Paper and pencils

Engagement activity

If you enjoy playing card games, discuss some of your favorites with the students. Then, ask students to share their favorite card games. Instruct students to examine their decks, taking note of the aces, number cards, and face cards. Note: You may wish to review the formula for calculating probability with students (i.e., the number of favorable outcomes divided by the number of total outcomes).

Hint: This activity can be completed individually or in groups of 2.

Exploration activity

1. Instruct students to count the number and face cards, then count the suits and the number of cards in each suit. Provide additional counting opportunities (e.g., count the 3s, count the queens). Have students record the totals for each item.

2. Ask the students to determine the probability of drawing each of the cards listed. Explain that they must give the answer, then convert it to lowest terms.

   - A face card (Answer: 12:52; lowest terms, 3:13)
   - A red card (Answer: 26:52; lowest terms, 1:2)
   - A spade (Answer: 13:52; lowest terms, 1:4)
   - A 7 (Answer: 4:52; lowest terms 1:13)
   - A 5 of clubs (Answer: 1:52; lowest terms, 1:52)

3. Ask students to explain how reducing the answers to lowest terms makes the probabilities easier to understand.

Evaluation

Have students complete problems similar to those used in the exploration activity (see the Card Predictors Worksheet, p. 324).

Card Predictors Worksheet
Answers

1. 2:52 = 1:23
2. 4:52 = 1:13
3. 2:52 = 1:23
4. 1:52
5. 4:52 = 1:13
6. 13:52 = 1:4
7. 2:52 = 1:23
8. 1:52
9. 12:52 = 3:13
10. 40:52 = 10:13

Extension activities

- Ask students to find the probability of drawing 2 or more specific cards from the deck.
- Ask students to find the probability of drawing specific cards from a pinochle or euchre deck.

Card Predictors Worksheet

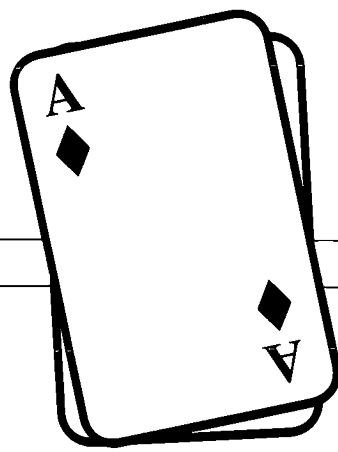
Name ______________________________

Instructions

Using a standard deck of playing cards, predict the following probabilities in lowest terms.

The probability of drawing:

1. A black 2 ______________
2. A queen ______________
3. A red queen ______________
4. A 2 of clubs ______________
5. A 9 ______________
6. A heart ______________
7. A black 4 ______________
8. A 4 of diamonds ______________
9. A face card ______________
10. A non-face card ______________



Probability Trees

About this learning activity

Students will use a tree diagram to identify possible menu combinations for a dinner and predict the probability that each will be served.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III X	V ○	VII ○
II X	IV ○	VI ○	VIII P

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Color tiles
- Paper and pencils

Spring Social Menu Options		
Main Entrée	*Side Dish*	*Beverage*
Chicken & noodles	Green beans	Coffee
Ham	Slaw	Soda
Hot dog	Potato chips	

Engagement activity

Facilitate a class discussion about students' favorite meals.

Exploration activity

1. Put the Spring Social Menu Options on the board or display it on an overhead projector.

2. Distribute color tiles to pairs or small groups of students. *(Hint: Use color tiles to represent each food item to help students solve this problem.)*

3. Instruct students to choose 1 item from each category in the menu to create a meal for the Spring Social.

4. Ask students to name the different meals they created. Draw a tree diagram on the board or overhead as they respond (see the Spring Social Tree, p. 329).

5. Facilitate a class discussion with these and other questions:

   ? How many meal combinations are possible? (Answer: 18)

   ? What strategies did you use to identify the various menus?

   ? Why does the tree diagram start with 3 branches? (Answer: There are 3 entrees—possible outcomes.)

   ? Why does each of these 3 split into 3 more branches? (Answer: There are 3 side dishes and 3 beverages.)

- ? What is the probability of being served slaw? (Answer: 3/18)
- ? What is the probability of being served each of the remaining menu items? Note: Encourage students to use the tree diagram to find the answers.
- ? How many branches are needed to expand the initial branches? Why? (Answer: 5 because it has 5 possible outcomes.)
- ? Do you observe any patterns in this tree diagram? Explain. (Answer: Yes, the first 3 possible outcomes are multiplied by the next 5 possible outcomes. Thus, 3 x 5 = 15.)
- ? How can you apply this pattern to similar problems? (Answer: Multiply the possible outcomes by each other.)
- ? What advantage(s) are gained by drawing a tree diagram instead of listing all the outcomes as they come to mind? (Answer: The tree diagram covers all the possibilities and avoids duplicating outcomes.)
- ? How can we relate these combinations to real life? (Answers will vary.)

Real-life applications

Ask students to solve this problem:

> *If you had 3 shirts and 4 pairs of jeans, how many different outfits could you make?* (Answer: 12)

Evaluation

Have students complete the School Carnival worksheet (p. 330).

Journal assignment
Tell students to describe the steps they would take to make a tree diagram. (Answer: Choose a letter to represent each possibility, decide how to combine possibilities, and draw branches to represent all possible outcomes.)

Extension activities

☆ Let students practice making tree diagrams for some or all of these situations:

- A pizza can come with thick, medium, or thin crust and pepperoni, cheese, mushrooms, green peppers, and/or onion toppings.
- A car can come in white, black, taupe, red, or blue and can include cruise control, luggage rack, tape player, power windows, and/or air conditioning options.
- A picture can be painted with acrylic, water color, or oil paints and painted in white, red, blue, orange, and/or yellow colors.
- Saturday night recreation choices include going to a movie, attending a ball game, or watching television at home and eating popcorn, pizza, and/or candy.
- A pair of jeans can be worn with tennis shoes, boots, track shoes, clogs, or sandals and a sweater, sweat shirt, blazer, and/or T-shirt.

Connections to other subjects

Art. Have students use empty toilet paper rolls and construction paper to make number shakers. Instruct them to decorate the shakers and cover the ends, leaving a small, wedge-shaped hole in 1 end. Tell students to add counters (e.g., beans, buttons, pennies) to their shakers. Have students pick a partner and use the completed shakers to play a probability game. (Adapted from *Math Art, Projects, and Activities.*)

Resources for teachers

Math Art, Projects, and Activities by Carolyn Ford Brunetto (Scholastic Professional Books, 1997)

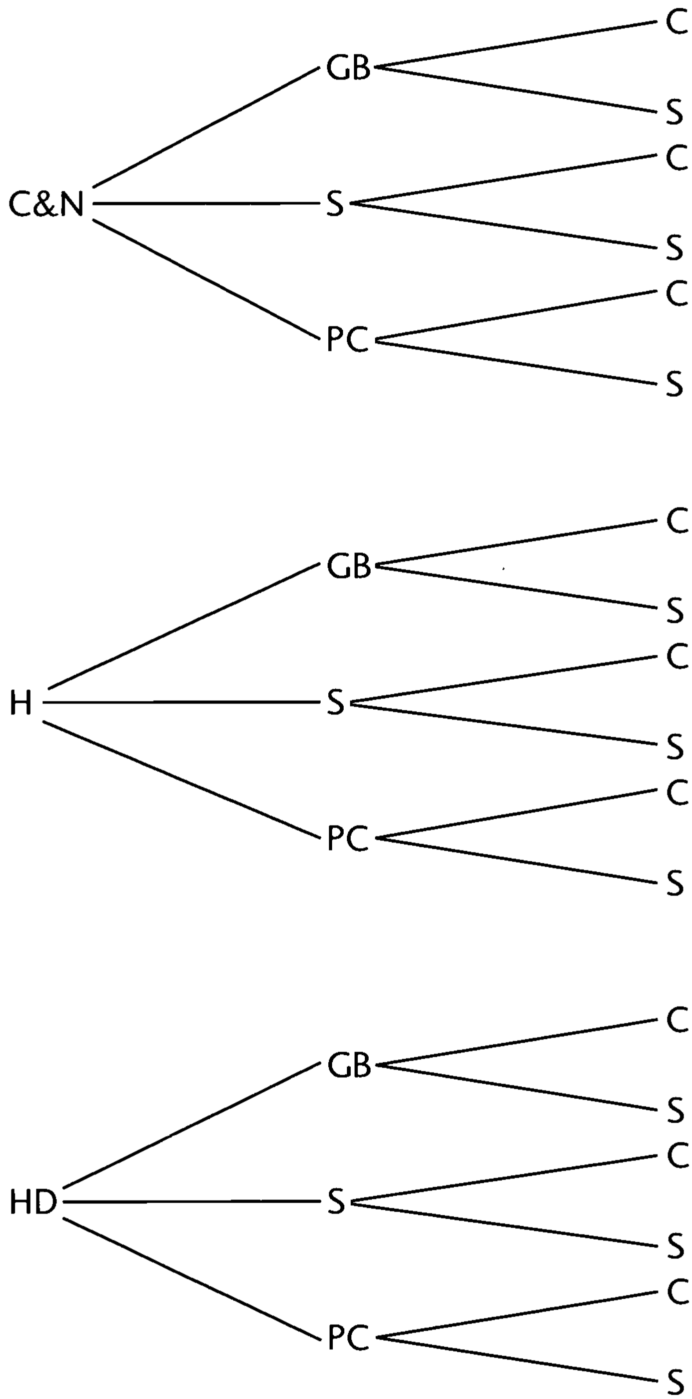
About Teaching Mathematics, A K–8 Resource by Marilyn Burns (Math Solutions Publications)

30 Wild and Wonderful Math Stories to Develop Problem-solving Skills by Greenberg (Scholastic Inc., 1992)

Math Games & Activities, Vol. 2, Toolbox of Duplicating Design for Middle and Upper Grade Elementary School Teachers by Paul Shoecraft (Dale Seymour Publications, 1984)

Hands-on Math! Ready-to-Use Games & Activities for Grades 4–8 by Frances M. Thompson (The Center for Applied Research in Education, 1994)

Spring Social Tree Diagram



School Carnival

Name ______________________________

Background

You can earn prizes at the school carnival by drawing a number from each of 2 boxes and adding the numbers together. Each box contains different numbers—Box A contains 1, 2, 3, and 4; Box B contains 5, 6, 7, and 8.

Instructions

1. On a separate sheet, draw a tree diagram to identify all possible combinations of numbers and sums.
2. Answer the following questions:

   a. What is the probability of getting a sum of 6?

   b. What is the probability of getting a sum of 9?

   c. What is the probability of getting a sum of 5?

   d. What is the probability of getting a sum of 7, 8, 10, 11, or 12?

   e. Which sum are you most likely to draw? Explain.

School Carnival Answers

a. 1 out of 16; 1/16

b. 4 out of 16; 4/16 or 1/4

c. 0 out of 16; 0/16

d. 16 out of 16; 16/16 or 1

e. You are most likely to draw the sum of 9 because there are more outcomes with the sum of 9 (4/16) than any other sum.

Road Racers

About this learning activity

Students will play a car racing game to practice probability and graphing skills.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II ○	IV ○	VI ○	VIII P

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Road Racer Game Boards (1 per group of 6 students)
- Game pieces (6 for each group)
- Die (2 for each group)
- Road Racers Activity Sheets (1 for each student)

Hint: A sample game board can be found on p. 338. You may wish to enlarge, color, and laminate your boards.

Engagement activities

Students could color the game boards as an engagement activity. You may wish to introduce or review principles of simple probability. Note: Students will develop an informal sense of probability as they play the game.

Exploration activity

1. Divide students into teams of 6 players and review the game rules. Note: The rules are outlined in the Explanation section.

2. Distribute the game materials to each team, and an activity sheet to each student.

3. Review Step 1 of the activity sheet with students. *(Hint: It is important for students to predict which racer will win before starting the game.)*

4. Instruct teams to play the game several times (i.e., 4–6) and record the winners in the chart (see Step 2 of the activity sheet). Then, have students finish their activity sheets.

5. As teams work, create a class chart on the board similar to the illustration.

	77	68	59	410	311	212
Group 1						
Group 2						
Group 3						
Group 4						
Group 5						
Total						

6. When all students have completed the activity, have teams report the total number of games each racer won. Record their data on the class chart that you drew on the board.

7. Have students total the racer columns. Record the correct sums on the class chart, then instruct students to create a bar graph that illustrates the class data.

8. Review the Die Combinations Table with students. Explain that this table illustrates the **theoretical probabilities** inherent in rolling 2 die.

9. Discuss students' answers to the activity sheet questions 5 through 7 as a class.

10. Instruct students to compare their winner predictions against the results. Facilitate a discussion that helps students identify methods for making the most accurate predictions.

Explanation

Road Racer Game Rules

1. Each player selects a racer number.

2. Players take turns rolling the 2 die.

3. A player moves 1 space (i.e., mile) according to the sum of the die, as listed:

   - A sum of 6 or 8 moves racer 68.
   - A sum of 5 or 9 moves racer 59.
   - A sum of 3 or 11 moves racer 311.
   - A sum of 4 or 10 moves racer 410.
   - A sum of 2 or 12 moves racer 212.
   - A sum of 7 moves racer 77.

   Note: A racer moves each time the die are rolled. The racer that moves is driven by the die sum, not by who rolls the die.

4. The first player to reach the finish line wins.

Key to theoretical probabilities used in the Road Racer Game

The sum of 2 = 1/36	Racer 212 = 2/36 (1:18)
The sum of 3 = 2/36	Racer 311 = 4/36 (1:9)
The sum of 4 = 3/36	Racer 410 = 6/36 (1:6)
The sum of 5 = 4/36	Racer 77 = 6/36 (1:6)
The sum of 6 = 5/36	Racer 59 = 8/36 (2:9)
The sum of 7 = 6/36	Racer 68 = 10/36 (5:18)
The sum of 8 = 5/36	
The sum of 9 = 4/36	Sums 2, 3, 4, 10, 11, 12 = 12/36 (1:3)
The sum of 10 = 3/36	Sums 5, 6, 7, 8, 9 = 24/36 (2:3)
The sum of 11 = 2/36	
The sum of 12 = 1/36	

Evaluation

Explain the instructions to the Sum Game:

> *The* ***Sum Game*** *is played with 2 die. A player agrees to take the sums 2, 3, 4, 10, 11, 12 and the other player agrees to take the sums 5, 6, 7, 8, 9. Players take turns rolling the die, scoring a point any time 1 of their sums is rolled. (A point is scored every time the die is rolled.) The first player to score 10 points wins the game.*

Have students predict which player is most likely to win the game and explain their answers using theoretical probability.

Journal assignment
Instruct students to write a journal entry explaining what they learned about probability by playing the Road Racer Game.

Extension activities

- Let students play the Sum Game, recording their data (i.e., number series chosen by each player, how many times each player wins) as they play. Have students graph the data and discuss the theoretical probabilities found.
- Have students change fractional probabilities to percentages.

Road Racers Activity Sheet

Name ________________________________

1. Before starting each game, predict which racer you think will win.

   Game #1 prediction ____ Game #2 prediction ____ Game #3 prediction ____

   Game #4 prediction ____ Game #5 prediction ____ Game #6 prediction ____

2. Play the game, putting a tally mark in the winning racer's column after each race.

Road Racers Game Chart

	77	68	59	410	311	212
Game 1						
Game 2						
Game 3						
Game 4						
Game 5						
Total						

3. Review your game chart. Based on the data collected, which racer has the best chance of winning? Smallest chance of winning? Explain.

4. Complete the Die Combinations Table by calculating the sum of the 2 die for each possible combination.

Your racer # is ________

Die Combinations Table

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

5. Shade or circle the combinations in the Die Combinations Table that allowed your racer to move.

6. With help from your teammates, complete the Probability Table.

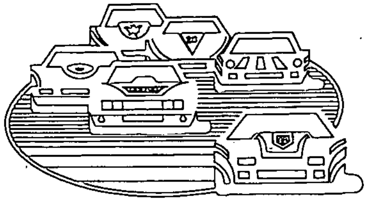
Probability Table

	311	59	212	68	77	410
Number of combinations that allow racer to move						
Probability of racer moving a space						

7. Using the Probability Table, rank the racers in order of their likelihood of winning.

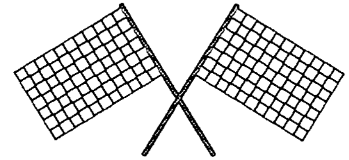
________ ________ ________ ________ ________ ________

Most likely to win … Least likely to win



Road Racer

Game Board



Start	Mile 1	Mile 2	Mile 3	Mile 4	Mile 5	Mile 6	Mile 7	Mile 8	Mile 9	Finish
77										
68										
59										
410										
311										
212										

358

357

Average M&Ms

About this learning activity

Students will sort and count M&Ms to calculate averages.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II ○	IV ○	VI ○	VIII P

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Hint: The students may use the worksheet or create and/or enter the data into a spreadsheet (e.g., Excel program).

Materials

- Fun-size packets of M&Ms (1 for each student) *(Hint: Skittles can also be used.)*
- M&M Worksheets (1 for each student)

Engagement activity

Tell students that the average grade on a mathematics test in a certain class was 84%. Ask them to explain what this factual statement means, using questions like:

? Does it mean that **everyone** scored 84%?

? Could some students have a higher grade? A lower grade?

Have students list other situations in which averages are used.

Exploration activity

1. Distribute the materials. Tell students to open their M&Ms packets and pour the contents carefully on their desks.

2. Ask for a show of hands, "How many have at least 2 **blue** M&Ms?" Have students explain why some packets do not contain 2 blue M&Ms.

3. Divide students into small groups, then instruct the teams to follow the instructions on the worksheet.

4. As teams complete the worksheet activities, have each appoint an accountant. Instruct the accountants to meet and combine their team totals to create class averages. Have them write their results on the board.

5. Discuss the class totals. Then, facilitate a class discussion with questions like:

   ? Did each group member's packet contain the same number of M&Ms?

   ? Did each group member's packet contain M&Ms of every possible color?

   ? How did your individual results compare with your team's results?

   ? How did your individual results compare with the class averages?

   ? How did your team's results compare with the class averages?

   ? What is an **average**?

300

Real-life applications

Have students apply averages to their daily lives by calculating the average hours they spend on common activities. Examples: Watching television at night and on the weekends, hours of sleep they get each week, and calories they eat each day.

Evaluation

Ask the students to average their grades for the last week or month. Then, have them identify the mean score.

Journal assignment

- Have the students describe the difference between individual average and class average.

- Ask students to answer these questions in their journals:

  - ? Was the number of M&Ms in your packet above or below the average?

  - ? What does it mean when something is described as above average? Below average?

Extension activities

- ☼ Have each student who has buttons on his/her clothing stand and count the buttons. Record each student's name and button total on the board. Have the class calculate the average number of buttons for the students with buttons, then for the entire class.

- ☼ Repeat the exploration activity, using multicolored cereals (e.g., Fruit Loops, Lucky Charms). Have students graph the results.

Connections to other subjects

Social Studies. Study the average incomes of families in the United States, in Japan, in China, and other countries.

Home connections

Ask students to have each member of their families record the hours they sleep for a week. Then, instruct each student to calculate the average hours of sleep for each family member and the average hours of sleep for the entire family.

M&M Worksheet

Name ______________________________

Instructions

1. Count the number of M&Ms in your packet. Write this number in the *Sample Size* column.
2. Sort and total the M&Ms in your packet by color, then record each color in its corresponding column.
3. Record the totals and colors of your teammates.
4. Calculate and record the **totals** for the data in your chart.
5. Calculate and record the **averages** for the data in your chart.

Team members	Sample Size	No. of Yellow M&Ms	No. of Brown M&Ms	No. of Orange M&Ms	No. of Blue M&Ms	No. of Green M&Ms	No. of Red M&Ms
1.							
2.							
3.							
4.							
5.							
6.							
7.							
Totals							
Averages							

Range, Mean, Median, and Mode

About this learning activity

Students will calculate the range, mean, median, and mode for several number series.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I X	III X	V ○	VII ○
II X	IV ○	VI ○	VIII P

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Paper and crayons or markers
- Grade Reports worksheets (1 per student)
- Transparency of Grade Reports Answers (p. 349)
- Overhead projector

Engagement activity

1. Ask 10 to 15 students to come to the front of the room.

2. Give each student a sheet of paper and a crayon or marker. Instruct students to write the number of letters in their last names. Example: Joe Smith would write 5 on his paper.

3. Arrange the students in a row, ordering them by their numbers (i.e., smallest to largest).

4. Point to both ends of the row and ask the class, "What is the difference between these 2 numbers?"

5. After students have answered, explain that the difference is called the **range**.

6. Ask the student with the most letters and the student with the fewest letters to sit down. (If 2 or more students have the same number, ask only 1 of them to sit down.) Have the class identify the range again.

7. Repeat the process until only 1 or 2 students remain standing. If only 1 student remains, explain to the class that this student's number is the **median** (or middle) number. If 2 students remain, explain that the average of their numbers is the median and have them compute the median number.

Hint: You may wish to review the concepts of range, mean, median, and mode with students prior to beginning this

Exploration activity

1. Distribute the Grade Reports worksheets and give the class ample time to complete it.

2. Put the worksheet answers on an overhead projector and help students review their answers. Encourage students to recalculate any incorrect answers and ask questions for clarification.

3. Facilitate a class discussion about the worksheet with questions like:

- ? What do you call the final grade in the Grade Reports worksheet? (Answer: Mean test score)
- ? What is another word for the mean? (Answer: Average)
- ? What is the median? (Answer: The middle number)
- ? In the engagement activity, what was the difference between the highest and lowest numbers called? (Answer: Range)
- ? In the engagement activity, a number appeared more often than the rest. When that occurs, what is the number called? (Answer: Mode)
- ? When finding the range, median, and mode, why is it important to order the data from least to greatest? (Answer: It is easier to identify the greatest and least numbers, repeating numbers are grouped together, and the middle number is easier to find.)
- ? When might a set of data not contain a mode? (Answer: If all the numbers occur the same number of times)
- ? Statistics involves collecting, classifying, and using numerical data. Can you name examples of statistics found in everyday life? (Answer: Weather data, city populations, batting averages)

Real-life applications

Help students identify ways that adults use the math concepts studied in this lesson. Examples: Calculating sports statistics and budgeting.

Evaluation

Have students complete the Range, Mean, Median, and Mode worksheet (p. 348). Answers to this worksheet can be found on p. 349.

Journal assignment
Have students define in their own words the terms range, mean, median, and mode.

Extension activity

☼ Have students work in groups of 4. Tell each group to choose a type of data they want to gather (e.g., time spent doing homework, talking on the telephone, or watching television in a week). Instruct the teams to collect data on their topics from classmates, then compile their findings and illustrate the results in bar graphs. Explain that teams must list the range, mean, median, and mode for their data under their graphs.

Connections to other subjects

Science. Have students use the diameters of the 9 planets and the sun as a number series. Instruct them to locate the diameters; list them from least to greatest; find the range; name 2 planets whose diameters are close in size; and name 2 planets whose diameters, when added, approximately equal the diameter of a third planet. Note: Students should use calculators for this activity.

Home connections

Tell each student to ask a family member to help him/her apply range, mean, median, and mode to a sports team's statistics. Instructions: Using a newspaper, select a sports team and analyze (depending on the season) the yardage gained per football game, basketball shooting percentages, or baseball batting averages. Then, chart or graph the data.

Resources for teachers

About Teaching Mathematics, A K–8 Resource by Marilyn Burns (Math Solutions Publications)

Hands-on Math! Ready-to-Use Games & Activities for Grades 4–8 by Frances Thompson (The Center for Applied Research in Education, 1994)

Math for Every Kid, Easy Activities that Make Learning Fun by Janice VanCleave (John Wiley & Sons, Inc., 1991)

Grade Reports

Name ________________________________

It is grade report time in Mrs. Baker's class. Help her find the mean test score for the past grading period. Round all grades to the nearest whole number. Then, answer the question at the bottom of the worksheet.

Language Arts	Test 1	Test 2	Test 3	Test 4	Test 5	Final Grade
Mary	84	76	87	82	88	
John	92	89	95	93	97	
Sue	79	77	82	84	76	
David	90	88	92	91	92	

Science	Test 1	Test 2	Test 3	Test 4	Test 5	Final Grade
Ellen	77	72	80	78	79	
Tim	83	84	89	88	90	
Joe	92	90	89	96	97	
Jim	98	96	95	99	100	

Math	Test 1	Test 2	Test 3	Test 4	Test 5	Final Grade
Jane	75	87	84	82	78	
Mark	80	89	78	79	85	
Jill	99	97	100	98	100	
Ted	86	100	93	100	96	

How would a sixth test for any or all subjects have changed the mean score?

Range, Mean, Median, and Mode

Name ________________________________

Instructions

Find the range, mean, median, and mode for each series of numbers.

(1) 80, 57, 69, 72, 88, 94, 29

Range _____ Mean _____ Median _____ Mode _____

(2) 49, 52, 47, 33, 27, 15, 51

Range _____ Mean _____ Median _____ Mode _____

(3) 81, 96, 25, 63, 84, 39

Range _____ Mean _____ Median _____ Mode _____

(4) 99, 84, 100, 42, 97, 85, 79, 95

Range _____ Mean _____ Median _____ Mode _____

(5) 100, 92, 93, 87, 98, 90, 83

Range _____ Mean _____ Median _____ Mode _____

(6) 76, 35, 49, 25, 66, 100, 32, 55, 96

Range _____ Mean _____ Median _____ Mode _____

(7) 3, 8, 9, 2, 5, 3, 8, 4, 3

Range _____ Mean _____ Median _____ Mode _____

(8) 6, 9, 8, 2, 9, 6, 4, 3

Range _____ Mean _____ Median _____ Mode _____

(9) 25, 100, 32, 55, 62, 99, 40, 36, 52

Range _____ Mean _____ Median _____ Mode _____

(10) 4, 7, 2, 9, 3, 6, 2, 2, 6, 8

Range _____ Mean _____ Median _____ Mode _____

Grade Reports Answers

Name	Final Grade	Name	Final Grade
Mary	83	Joe	93
John	93	Jim	98
Sue	80	Jane	81
David	91	Sally	82
Ellen	77	Jill	99
Tim	87	Ted	95

Having a sixth test would have changed each student's mean score, because the sum of the scores would be divided by 6 instead of 5.

Range, Mean, Median, and Mode Answers

(1) Range: 65	Mean: 70	Median: 72	Mode: none
(2) Range: 37	Mean: 39	Median: 47	Mode: none
(3) Range: 71	Mean: 65	Median: 72	Mode: none
(4) Range: 58	Mean: 85	Median: 90	Mode: 85
(5) Range: 17	Mean: 92	Median: 92	Mode: none
(6) Range: 75	Mean: 59	Median: 55	Mode: none
(7) Range: 8	Mean: 6	Median: 8	Mode: 8
(8) Range: 7	Mean: 6	Median: 6	Mode: 6
(9) Range: 75	Mean: 56	Median: 55	Mode: 55
(10) Range: 7	Mean: 5	Median: 5	Mode: 2

Graphing the Weather

About this learning activity

Students will collect weather-related data, illustrate it in a graph, and use the graph to answer questions about the data.

Process skills

○ Building models
○ Categorizing or classifying
○ Communicating
○ Comparing
○ Controlling variables
○ Experimenting
○ Hypothesizing
○ Inferring
○ Interpreting data
○ Measuring
○ Observing
○ Ordering
○ Predicting
○ Reasoning
○ Recognizing relationships
○ Recording

Ohio mathematics proficiency strand(s)

I ○	III X	V ○	VII ○
II X	IV ○	VI ○	VIII P

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Magazines (e.g., *U.S. News & World Report, Time, Newsweek*)
- Newspapers (e.g., *USA Today*)
- Index cards
- Scissors
- Glue
- Peg board (20 x 20 holes)
- Masking tape
- Yarn
- Golf tees
- Colored markers
- Colored pencils
- Dot paper (provided on p. 355)

Engagement activity

Have students work in groups to locate and cut out different types of graphs from magazines and newspapers. Instruct them to glue their graphs to index cards, then sort the cards according to graph type (e.g., line graph, bar graph, pie chart, pictograph). Provide a place for groups to display their index cards by category. After students have reviewed all of the cards, help them identify the graph categories and discuss the uses for each. Be sure that these types of graphs are recognized during the discussion: circle, line, bar, and pictograph.

Exploration activity

1. Create "pegboard graph paper" with these steps:

    a. Place a strip of masking tape along both the horizontal and vertical axes of the pegboard. Place the pegboard on the chalkboard tray or hang it on a bulletin board.

    b. Use the index cards and markers to prepare labels for horizontal and vertical axes. Place the cards at every other hole.

    c. Prepare a title card (e.g., Daily Temperatures) for the graph and attach it to the top of the pegboard.

Hint: You may wish to let students help with steps a–c.

2. For 5 consecutive days, have teams of student volunteers (a different team each day) create a line graph by following these steps:

    a. Measure the temperature outside the school.

    b. Record the data on an index card.

    c. Place the card at the appropriate interval on the pegboard.

    d. Place a golf tee in the corresponding hole.

    e. Wrap a long piece of yarn around the golf tee. Note: The yarn will be used to connect the tees together in the order in which they are placed. Be sure the yarn is kept taut.

3. After the data has been placed on the graph for the fifth day, give students colored pencils and dot paper. Have them copy the pegboard line graph on the dot paper, labeling the axes, title, and yarn paths with red pencil.

4. Divide students into groups to discuss and record answers to these questions:

    ? Which day was the temperature the highest? Lowest?

    ? What was the difference in temperatures between Day 1 and Day 5?

    ? On which 2 consecutive days was the temperature the highest? Lowest?

5. Facilitate a class discussion about the pegboard line graph with questions like:

    ? What kind of graph is this? (Answer: A line graph)

    ? What can a line graph illustrate? (Answer: Change in data over time)

    ? Would a line graph be the best type to use in displaying the heights of your classmates? Explain. (Answer: No, because there is no change in the data.)

    ? What kind of graph would work better for displaying heights? (Answer: A bar graph)

    ? What other kinds of data could be represented on a line graph?

    ? How does a line graph show an increase over time? A decrease? No change? (Answer: The line slopes up for increase, down for decrease, and remains level for no change.)

? How might you use a circle graph when illustrating weather? (Example: The number of days in a year that fall within certain temperature ranges)

? What steps did you use to copy the pegboard graph on dot paper? (Answer: Drew a graph, collected data, marked and connected points, titled the graph)

Real-life applications

Have students review and discuss the graphs they cut out for the engagement activity. Help them identify the ways each graph is used by adults for business and/or home management purposes.

Evaluation

Give students dot paper, colored pencils, and the following information: *Katherine took a spelling test once a week for 5 consecutive weeks. She scored 76, 79, 82, 95, and 89, respectively.* Instruct them to make a line graph, then answer these questions:

? Which week did Katherine score the highest grade? The lowest grade?

? Did her scores increase or decrease over time?

? Which week's score might indicate that Katherine watched television instead of studying her spelling?

Journal assignment
Have students describe the steps they would use to create a line graph, a bar graph, and a circle graph.

Extension activity

☼ Have students choose 2 types of graphs from those pasted on cards for the engagement activity. Instruct them to write at least 3 higher-level questions (i.e., requiring computations) that can be answered by using the graph. Have students write 3 questions for each graph type. Then, let students work in pairs to exchange, answer, and check each other's questions.

Connections to other subjects

Science. Have students contact local water companies to gather community water consumption statistics. Tell them to graph the data and look for trends in the community's water usage.

Social Studies. Instruct students to use an almanac to collect data on population trends over a given period of time, then make line graphs and develop questions to exchange with other classmates.

Home connections

Ask family members to give students the gas or electric bills from the previous calendar year. Instruct students to graph the monthly costs and identify trends in their families' energy usage. The type of graph used will depend upon the types of trends students choose to illustrate; confirm that students made the correct graph choices.

Resources for teachers

About Teaching Mathematics, A K–8 Resource by Marilyn Burns (Math Solutions Publications)

Hands-On Math! Ready-to-Use Games & Activities for Grade 4–8 by Frances M. Thompson (The Center for Applied Research in Education, 1994)

Math for Every Kid, Easy Activities that Make Learning Fun by Janice VanCleave (John Wiley & Sons, Inc., 1991)

Dot Paper

Water Graph

About this learning activity

Students will collect and record water temperatures at regular intervals, then graph their data.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II ○	IV ○	VI ○	VIII P

Note: The primary strand is marked with a "P" and each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Water
- Ice (6–8 small cubes per team)
- 1-quart containers (1 per team)
- Thermometers (1 per team)
- Stopwatches or timers (1 per team)
- Water Temperatures worksheets (1 per student)
- Graph paper (You can copy the graph paper on p. 360.)

Engagement activity

You may wish to have students participate in setting up the water temperature experiment (e.g., gather materials, distribute them).

Exploration activity

1. Organize students into teams and distribute the materials (1 set per team). If necessary, review the instructions on the Water Temperatures worksheet.

2. Instruct students to complete the experiment. Monitor students as they work.

3. Have students share their graphs. Help them compare and contrast any differences.

4. Ask students to estimate future temperature readings, explaining their predictions.

Explanation

As the ice melts, the water temperature drops until it levels off. If the room is quite warm, the water temperature may rise after all of the ice melts until the room temperature and the water temperature reach a state of equilibrium.

Real-life applications

Explain that graphs are used frequently in business to identify trends (e.g., increases, decreases, remaining the same). Help students list situations in which knowledge of trends would be helpful. Examples include tracking product defects, monitoring accidents, and identifying the most common customer complaints.

Evaluation

Ask students to gather some data and illustrate it in a graph. Let the students choose whether to make a bar graph, circle, or line graph and explain their choices.

Journal assignment
Ask students to explain how they choose the best way to present data that they collect (e.g., data chart, line graph, circle or pie graph, bar chart).

Extension activity

- Instruct students to take an automobile color sampling by recording the number and color of cars that pass the school during a particular time period. Have the students illustrate their sampling in a **chart**, then convert the chart into a graph. Note: Students should choose a bar graph.

Connections to other subjects

Social Studies. Have each student gather population information for the past 10 years for a particular city, town, or state. Let students choose their own methods for presenting the data to the class.

Physical Education. Instruct students to name their favorite sports. List the sports on the board, using tally marks for repetitions. Have students create a circle graph that illustrates the percentage of students that prefer each sport.

Resources for teachers

You can obtain data for students to use in practicing graphing skills from the U.S. Census Bureau at the following website: **http://www.census.gov**

Water Temperatures

Name ______________________________

Instructions

1. Fill the container half full with water. Place the thermometer in the water.
2. Read the thermometer to find and record an initial temperature reading on the Water Temperatures Data Sheet. Note: Readings can be taken in either Fahrenheit or Celsius. Also, practice using your time piece before placing the ice in the water.
3. Put all of the ice cubes in the water. Stir gently.
4. Take and record temperature readings every 15 seconds for 3 minutes. Stir the water between readings.
5. Plot the data on the graph paper. Draw the best possible curve through the plotted points.

Water Temperatures Data Sheet

	Temperature measured in degrees Fahrenheit	Temperature measured in degrees Celsius
Initial reading		
15 seconds		
30 seconds		
45 seconds		
60 seconds		
1 minute, 15 seconds		
1 minute, 30 seconds		
1 minute, 45 seconds		
2 minutes		
2 minutes, 15 seconds		
2 minutes, 30 seconds		
2 minutes, 45 seconds		
3 minutes		

One-Inch Graph Paper

Fun with Math: Real-Life Problem Solving for Grades 4–8

Multi-strand Activities

This section contains learning activities that integrate strands, allowing students to practice math skills in less-isolated situations. In addition, lessons are comprehensive and complex. Thus, the multi-strand activities give students opportunities to solve mathematical problems in ways that are similar to those encountered in real-life situations.

Learning Activities **Page**

Notes

Clock-wise

About this learning activity

Students will build clocks, then use the hands on their clocks to illustrate various angles.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III X	V ○	VII ○
II X	IV X	VI X	VIII ○

Note: Each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

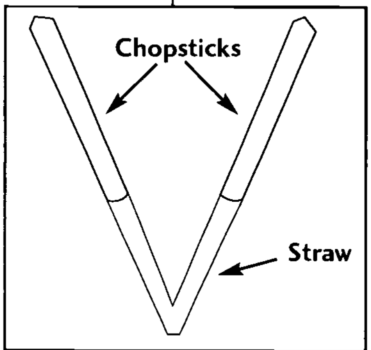
○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Chopsticks (2 per student)
- Straws (1 per student)
- Clock patterns (see p. 366)
- Brass fasteners (1 per student)
- Scissors (1 per student)
- Demonstration clock (optional)

Engagement activity



Demonstrate how to construct a chopsticks angle device from 2 chopsticks and 1 straw, as illustrated. Then, distribute the chopsticks and straws, and have students construct the angle device. Ask students to use their devices to illustrate types of angles as you call them out: acute, right, obtuse, straight, 80°, and other angle degrees. For example, say, "Show me 80°." *Hint: Encourage students to use 90° and 180° as baselines to help them demonstrate the other angle degrees.*

Exploration activity

1. Distribute clock patterns, scissors, and brass fasteners. Instruct students to cut out the clock and hands, then construct the clocks.

2. Challenge students to demonstrate angle and time concepts with questions like:

   - Can you make a 90° angle with the clock hands?
   - What time is it?
   - Do other clock times also look like a 90° angle?
   - What clock time looks like a 30° angle? Explain. *(Hint: This is a good opportunity to teach 360°, i.e., 12 hours in a day = 30°.)*
   - How many clock times look like 180°?
   - What clock time causes the hands to form a diameter? A radius? A 20° angle?

Evaluation

Provide several open-ended questions to evaluate student learning. Example: How would you determine the number of degrees that correspond to 1 minute on the clock? Show and explain your work.

Journal assignment
Have students draw a picture of the angle shown on their clocks when they get up in the morning.

Extension activities

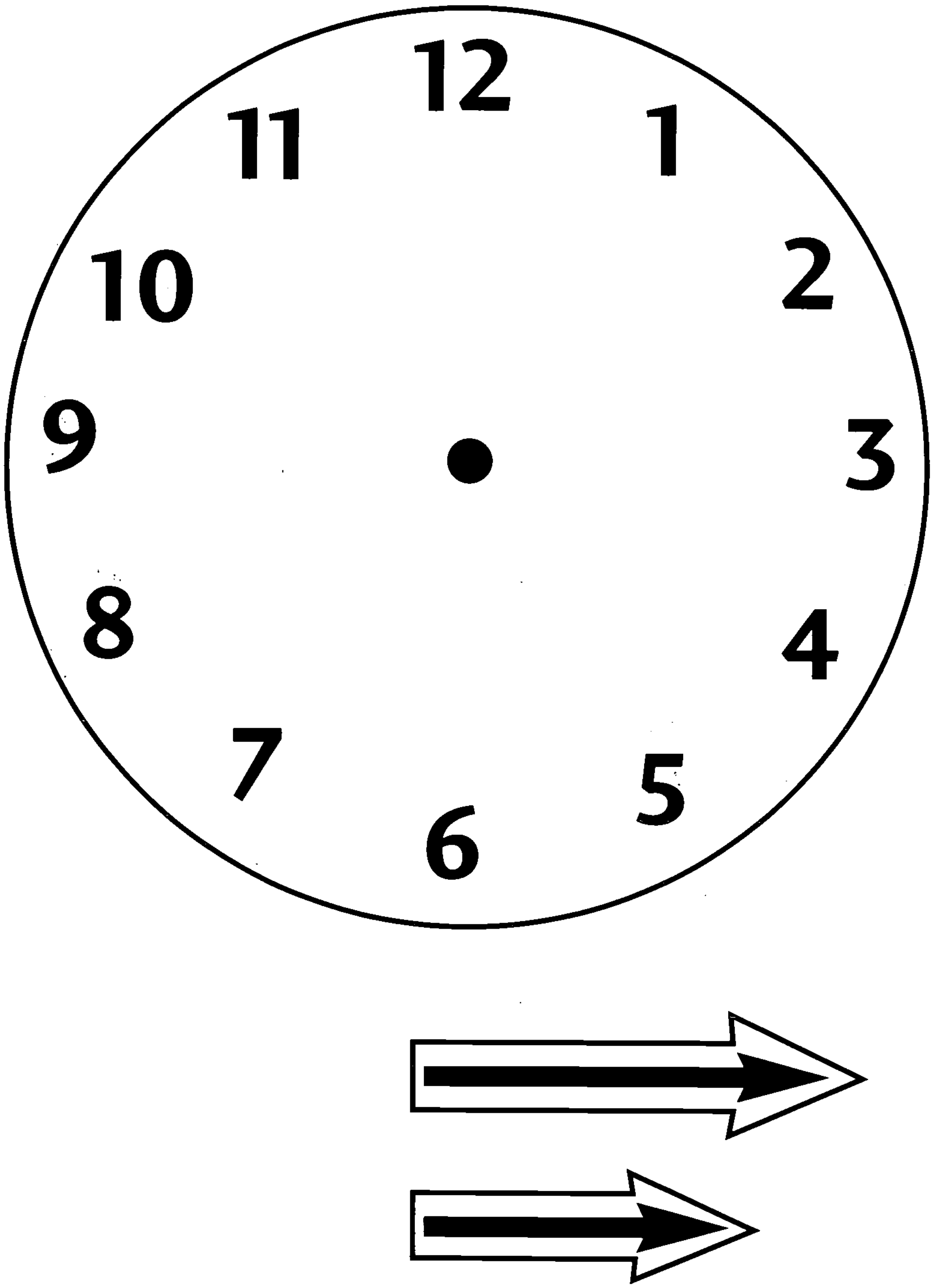
- Provide opportunities for students to relate degrees and parts of the clock to fractional parts of a whole. Example: 15 minutes (12:15 p.m.) = 90° = 1/4 of an hour.
- Have students convert percents used in circle graphs to angle degrees.

Home connections

Tell students to work with family members to answer the listed questions. What angle do the hands on the clock represent when:

- You sit down to eat dinner?
- Your favorite television program is on?
- It is time for you to go to bed?

Clock Pattern



Snack Pack Analysis

About this learning activity

Students will use packages of snacks to collect data and give reports.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III X	V ○	VII ○
II X	IV X	VI X	VIII X

Note: Each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials (1 per student)

- Variety package of snacks (e.g., corn chips, potato chips, cheese curls)
- Measuring device (e.g., measuring tape, meter stick, rulers, balance scales)
- Paper towels
- Graph paper
- Markers or colored pencils (optional)

Engagement activity

Tell students the following story:

> *Demetrius and Megan were comparing bags of chips in their lunches when Megan noticed she had fewer chips than Demetrius. Feeling cheated, she went home and shared the situation with her mother. Megan's mother called Demetrius' mother and the 2 moms decided to call the manufacturer. They explained the situation to a customer service representative, who referred the complaint to a quality control manager. She decided to analyze the problem by collecting data, then report her findings to the appropriate people.*

Explain to students that they will be helping the quality control manager with her analysis and report.

Exploration activity

1. Distribute the materials, then organize the students into teams according to type of snack.

2. Instruct students to analyze the customers' complaint, then summarize their findings and make suggestions for solving the problem in a report. Explain that they can choose their own analysis and reporting methods. **Emphasize that everyone on the team must participate.**

3. Let groups present their reports, while the rest of the class role plays a typical reaction by a customer and/or company executive.

> Note: Be sure to set and enforce ground rules that maintain an atmosphere of constructive criticism, civility, and fun.

4. When all groups have presented, facilitate a class discussion with questions like:

   ? Did the reports show a reasonable problem analysis? Why or why not?

   ? Did groups show a sincere desire to solve the customers' problem? Explain.

   ? Which solutions showed the most creativity? Why?

   ? Why is creativity important to the problem-solving process?

Explanation

This activity is open-ended to allow students to initiate a variety of problem-analysis and problem-solving methods. Possible strategies include:

- Count the contents.
- Measure the total length by placing the snacks end to end.
- Calculate the average number of snacks per bag.
- Illustrate the package contents in a bar graph.
- Measure the bag weight.

Evaluation

Prior to starting this lesson, have the class help you create an assessment rubric. Sample criteria:

- How well does the group demonstrate an understanding of the task assigned?
- Is the report complete (i.e., data, graphs, and suggestions for the quality control manager)?

Journal assignment
Have students describe the problem-solving procedures they used to complete the Snack Pack Analysis.

Extension activities

- Repeat the activity, using a different brand of snacks. Then, compare and contrast the results.

- Write a letter to the snack company, presenting the class's findings and suggestions. In addition, you may wish to have students write a letter to praise a company that is doing a good job.

Connections to other subjects

Language Arts. Have students write a letter to 1 or more companies, as suggested in the Extension activities section.

Social Studies. Help students relate their learning in the Snack Pack activity to the economics of business practices.

Home connections

Guide students toward recognizing that skills in mathematics and problem solving will help them become wise consumers by asking family members to help students participate in some comparison shopping (e.g., compare package volumes and contents to costs) at the grocery store.

Resources for teachers

The Frito-Lay website: **www.fritolay.com/faqs.html**

Volume Validity

About this learning activity

Students will use both a mathematical formula and water displacement to measure the volumes of objects.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII ○
II X	IV X	VI X	VIII X

Note: Each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Cm cubes (minimum 36 per team) *(Hint: Interlocking cubes work best.)*
- Graduated cylinders (50 ml or 100 ml)
- Beakers (to hold water)
- Irregular objects (e.g., pencils, markers, erasers, magnets, nuts, bolts, nails)
- Volume Validity worksheets

Engagement activities

1. Tell students the story of Archimedes and the King's crown. Point out how it was discovered that the King's crown was not 100% gold. Help students recognize what Archimedes discovered—when an object is submerged in liquid, it **displaces** an amount of the liquid equal to its volume, thereby allowing the object's volume to be measured. Explain that students will test Archimedes' discovery as they use water displacement to measure volume. Help students define **volume** and how it is measured. (Answer: The amount of space an object or substance occupies, measured in cubic units like cubic cm and ml.)

2. Organize students into teams and distribute the cm cubes. Have students practice making rectangular prisms with the cm cubes while you set up and distribute the remaining materials.

Exploration activity

Part I: Volume by formula

1. After students become familiar with the concept of volume in the engagement activity, discuss how to calculate volume, and its relationship to area.

Volume = L x W x H

Area = L x W

2. Help students discover that volume can be found by counting the number of cm cubes required to construct their prisms too.

3. Demonstrate **calculating volume** with the formula by completing the first row in the worksheet chart as a class. Have teams build a 2 x 1 x 3-cm prism. Tell them to review how the dimensions are recorded on the worksheet, then calculate the volume.

4. Students should ask how they can determine which dimension is L, W, H. (If no one asks, encourage such a question.) Explain that it depends on the orientation and use a prism to illustrate the possible orientations of a 2 x 1 x 3-cm prism. Stress, however, that to calculate the volume correctly, students must be **consistent** in accounting for all 3 dimensions.

5. Ask teams to build another prism with the **same volume** as the 2 x 1 x 3-cm prism. Discuss their findings.

6. List these sizes on the board: 3 x 2 x 4 cm, 2 x 2 x 2 cm, 1 x 1 x 1 cm, 2 x 2 x 6 cm, 3 x 2 x 3 cm, 5 x 2 x 1 cm, 1 x 2 x 4 cm. Tell teams to build 7 more prisms in the sizes listed. Instruct them to record the dimensions and calculate the volume for each prism on the worksheet chart.

7. When teams finish working with the 7 prisms, let them compare results and work out any discrepancies among themselves.

Part II: Volume by displacement

1. Have teams display a 1 x 1 x 1-cm prism and find its volume in their charts. Ask students, "What do you think will happen if I drop this 1 x 1 x 1-cm prism (1 cube) into a graduated cylinder containing 20 ml of water?"

2. After students predict outcomes, instruct teams to test the predictions. Note: Have teams pour 20 ml of water from their beakers into their cylinders.

3. Ask teams to report the results. (Answer: The water level rose 1 ml.)

4. Instruct teams to repeat the experiment by adding first 2, then 5 more cubes (total of 8 cubes). Facilitate a class discussion with questions like:

   ? Where should the water level be now? [Answer: The water level should be 8 ml from the beginning water level (20 ml) or 28 ml.]

   ? From this experiment, what could you say about the relationship between 1 cubic cm and 1 ml? [Answer: One cubic cm **displaces** 1 ml of water (1 cubic cm = 1 ml).]

   ? Can you identify a formula for calculating volume by displacement. [Answer: V = NL – OL, where NL = the new water level and OL = the original water level (ml).]

? How would you record a 1 x 1 x 1-cm cube's volume on your charts? (Answer: Record the amount of water displacement in the last column.)

5. Have students complete the 1 x 1 x 1-cm cube row in their charts.

6. Instruct teams to use the **displacement method** to calculate the volume of the remaining prisms listed on their charts (i.e., drop the other prisms constructed in Part I into the graduated cylinder).

7. Help students compare the values recorded in Part II (displacement method) with those recorded in Part I (formula method) of the activity.

Part III: Volumes of irregular objects

1. Explain to students that thus far all of their objects have been considered "regular" because they were modeled directly after prisms. Display some "irregular" objects, explaining why they are known as *irregular*.

2. Help students recognize that the formula method cannot be used to determine the volume of irregular objects. Have them explain why. Then, ask students if they think the displacement method will work.

3. After students share their predictions, give each team at least 2 irregular objects and challenge them to calculate the volumes using the displacement method. Instruct them to record their findings in the last **rows** of their charts, then answer the 2 questions at the bottom of their worksheets.

4. Discuss the results of the final experiment and the students' answers to the worksheet questions.

Evaluation

Provide additional objects (both regular and irregular), and have students choose a method and determine their volumes.

Journal assignment
Have students complete 1 or both of these sentences:

- To find the volume of an orange, I would ______________________.
- Two ways to find the volume of a brick are ______________________.

Extension activities

- Have students determine the number of cm cubes it would take to model the volume of a 2-liter bottle.

- Introduce the concept of density—**density** = $\frac{\textbf{mass}}{\textbf{density}}$

  Demonstrate how to determine density, including the densities of objects of equal volume and different masses. Have students build and mass objects of sizes similar to the prisms they built in the exploration activity, then compare densities.

Connections to other subjects

Economics. Have students compare the economic uses of the word *volume* (e.g., large volume of goods or customers) to the mathematical definition.

Home connections

Instruct students to identify common water displacements in the home (e.g., getting into a bathtub, placing eggs in a saucepan of water for boiling).

Volume Validity

Name ______________________________

Object	L	W	H	Volume by formula	Volume by displacement
2 x 1 x 3	3	2	1		
	XXXXXXXX	XXXXXXXX	XXXXXXXX	XXXXXXXX	
	XXXXXXXX	XXXXXXXX	XXXXXXXX	XXXXXXXX	
	XXXXXXXX	XXXXXXXX	XXXXXXXX	XXXXXXXX	

List 2 ways to write cubic cm: 1. __________ 2. __________

What does *water displacement* mean?

The Giant Mystery

About this learning activity

Students will choose and apply mathematical problem-solving methods to estimating the size of a giant.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III X	V ○	VII X
II X	IV ○	VI X	VIII ○

Note: Each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Hint: Instructions for creating the ***giant's top hat*** can be found on p. 381. Make the hat prior to starting the activity.

Materials

- Various measuring devices (e.g., measuring tapes, rulers, string)
- Calculators (optional)
- Giant's top hat

Engagement activity

Create a relevant story to drive the giant theme. A sample is provided, please customize it to fit your circumstances or use it as a model for your own story.

> *Lauren, Dustin, Rafael, and Sonja were enjoying the log ride at King's Island Amusement Park when they noticed a large figure in the woods. They pointed and screamed in amazement, "Look, look, its huge!" When they exited the ride, they told others what they saw, but nobody believed them. They decided to board the log ride again, hoping to see the figure. And they did! Rafael reached out and grabbed the creature's hat—**a giant top hat**.*

Now display the giant's top hat.

Exploration activity

1. Organize students into teams of 2–3. Challenge teams with this question:

   > *Based on the information provided in the story, how can Lauren, Dustin, Rafael, and Sonja give park officials an accurate description of this giant's size?*

   Note: Write the question on the board so students can refer to it during the activity.

Hint: Let students borrow the hat, ***if they request***

2. Instruct students to work together in their teams to answer the question. Tell them to use any measuring devices they choose from those available. Explain that they can use calculators too.

3. **Important!** Give students ample opportunity to find their own methods for solving this problem. Some students may need help getting started. Encourage them to use words, pictures, and symbols to help them think. If students continue to struggle or get stuck, use questions like those listed in Step 4 to stimulate thinking.

4. As teams finish their problem-solving processes, facilitate a class discussion with questions like:

   - ? What methods did you use to determine the giant's height? (Have teams describe them.)
   - ? Could the giant stand up in this classroom? Why or why not?
   - ? If you asked the giant to stretch out on the floor, where would his head and feet rest?
   - ? How can you assess the accuracy of your estimate of the giant's size?

Explanation

Problem-solving processes and solutions will vary. Limit the amount of information given at the beginning; left to their own devices, students will eventually tap into prior knowledge and skills.

Some students may use proportional reasoning to solve the problem. For example, the ratio of head circumference to height is approximately 1:3. The ratio of shoe length to height is approximately 1:6. If a student's height is approximately 3 times the circumference of her head and the giant's head is 60 in around, then the student can multiply 60 x 3 to find the approximate height of the giant.

Students will likely find other methods to determine the giant's height. Encourage their creativity.

Evaluation

Instruct students to collect head circumference and height data for everyone in the class, calculate the class averages for each, and use the data to check the accuracy of their estimates of the giant's height. Ask students, "Did using additional data change the estimates? Explain."

Journal assignment
Have students answer these questions in their journals:

- ? How did you feel about the problem-solving process? Finding the solution?
- ? What mathematical methods and skills did your group use?

Extension activities

- ☼ Have students calculate the size of the giant's hand or foot.
- ☼ Give students additional problems that require proportional reasoning.

Connections to other subjects

Language Arts. Have students write stories similar to the giant story that would allow them to calculate size.

Home connections

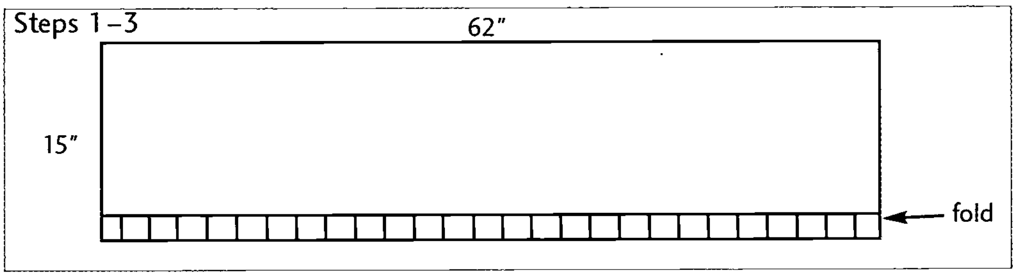
- Have students practice proportional reasoning by comparing amounts of ingredients to number of servings.
- Instruct students to measure heights and shoe lengths of family members, then calculate ratios for each.

How to Make the Giant's Top Hat

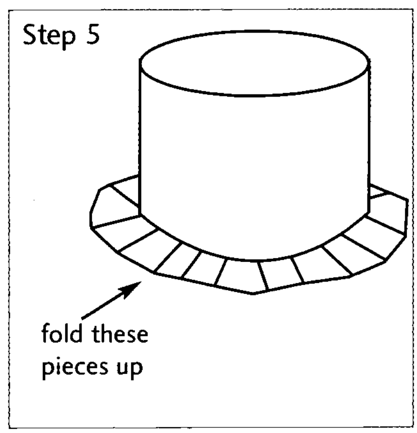
Materials

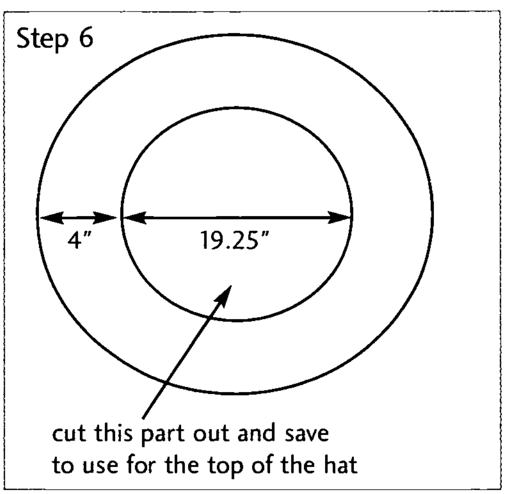
- Poster board or heavy construction paper
- Pencil
- Scissors
- Glue and/or tape
- Ruler

1. Cut a piece of poster board or construction paper 62 in long and 15 in wide.



2. Measure 2 in from the bottom and draw a line across the length of the 62 x 15-in piece. Fold on the line, then open again.
3. Cut several slits vertically along the 2-in strip, as illustrated.
4. Roll the piece lengthwise to form a cylinder. Let it overlap no more than 2 in. (Ideally, the hat's circumference should be 60 in.) Glue or tape the ends together, leaving the folded and cut end free.
5. Fold up the strips along the bottom of the hat to form a ledge for the brim.





6. Draw and cut a round piece (23.25 diameter) from the poster board, as illustrated. Cut the top of the hat from the center, leaving the brim. *Hint: You may wish to cut the hat top and brim with a box knife for ease of cutting and to keep each in a single piece.*
7. Slide the brim carefully over the hat so that it rests on the ledge. Glue or tape it in place.
8. Glue or tape the hat top in place.
9. Paint or decorate the hat (optional).

Speedy

About this learning activity

Students will use problem-solving strategies and mathematical skills to identify the best choice between 2 options.

Process skills

○ Building models
○ Categorizing or classifying
○ Communicating
○ Comparing
○ Controlling variables
○ Experimenting
○ Hypothesizing
○ Inferring
○ Interpreting data
○ Measuring
○ Observing
○ Ordering
○ Predicting
○ Reasoning
○ Recognizing relationships
○ Recording

Ohio mathematics proficiency strand(s)

I X III ○ V ○ VII ○
II X IV ○ VI X VIII X

Note: Each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Graph paper
- Chart paper
- Markers or colored pencils
- Calculators

Engagement activity

Review speed limits for your local area with students (e.g., 25 mph on residential streets, 20 mph in school zones, 55 mph on state routes, 65 mph on federal highways). Then, discuss what could happen if a driver is caught exceeding the speed limit, including average speeding ticket costs.

Exploration activity

1. Present this real-life scenario to students:

   Howard Riley was speeding down State Route 99. He was traveling 80 mph in a 55 mph zone. Hearing a siren, he glanced in his rearview mirror to find a sheriff's car. He quickly pulled over to the side of the road and awaited "the damage." The Sheriff checked Mr. Riley's driver's license and proof of insurance, then explained that the county is testing a new program that allows citizens to choose their penalty for speeding from 2 choices:

   Option A. Pay a $10.00 administrative fee, plus $4.00 per mile for each mph over the posted speed limit.

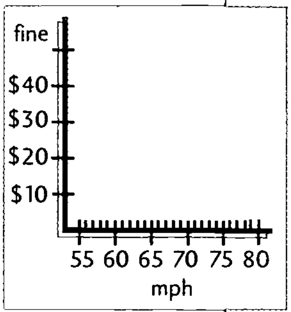
   Option B. Pay $5.00 per mile for each mph over the posted speed limit with no administrative fee.

Hint: You may wish to customize the scenario to your situation or create your own.

2. Instruct students to determine which option Mr. Riley should choose. Note: Read the story again so that students can record the facts, if needed.

3. Discuss students' answers, having them explain their methods.

4. Give students different speeds (e.g., Mr. Riley was traveling 63 mph). Several scenarios can be found in the Speedy Solutions table, p. 386.



5. Help students recognize that numerical parameters exist for each option. Have them use their solutions (e.g., fines calculated) to create a graph which will help them find the parameters. [Answer: The graph will illustrate mph (independent variable) vs. money (dependent variable).]

6. Facilitate a class discussion about the speeding ticket options with questions like:

   ? Which option offers the best deal for the driver who is exceeding the speed limit by 10 mph or less?

   ? What happens as the amount by which he exceeds the speed limit increases?

   ? What is the least amount someone could be fined under the trial system? The greatest amount?

   ? How much would Mr. Riley pay if he was traveling 75 mph and chose Option A? Traveling 100 mph and chose Option B?

   ? What does the graph illustrate?

   ? Can you think of another scenario that would have a similar graph?

Explanation

If students are experiencing problems with where to begin, suggest that they record their data in a table like the Speedy Solutions table on p. 386.

Evaluation

Have students create similar problem scenarios. Assess their graphing, computation, and reasoning skills.

Journal assignment
Tell students to name a problem-solving strategy they learned during this activity and explain why it is useful.

Extension activities

- ☼ Instruct higher-skilled students to determine the equation of the lines by finding slopes and intercepts. Note: Students could use graphing calculators, if available.
- ☼ Ask students to modify the scenario and repeat the activity. Modification examples include making the fine dependent upon the make and model of the car or the time of day.
- ☼ Have students solve algebra equations related to the scenario. Example: Find "n" mph in equations that provide the amount of the fine and the speed limit.

Connections to other subjects

Social Studies. Have students study the history of speeding tickets and how they have changed over time. Instruct them to collect and graph data concerning how much citizens spend for speeding tickets per year, per month, per geographic location, and similar facts.

Home connections

Ask family members to share with students their experiences with choosing between 2 options and the criteria they used to make their decisions.

Speedy Solutions

Miles per hour (mph)	Option A	Option B
55	$ 0	$ 0
56	14	5
57	18	10
58	22	15
59	26	20
60	30	25
61	34	30
62	38	35
63	42	40
64	46	45
65	50	50
66	54	55
67	58	60
68	62	65
69	66	70
70	70	75
71	74	80
72	78	85
73	82	90
74	86	95
75	90	100
76	94	105
77	98	110
78	102	115
79	106	120
80	$110	$125

Tower Power

About this learning activity

Students will use mathematical knowledge to construct a tower that will support the weight of a ping-pong ball.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III X	V ○	VII ○
II X	IV X	VI X	VIII X

Note: Each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Paper and pencils
- Plastic drinking straws
- Masking tape
- Stale (hard, dried out) marshmallows
- Toothpicks
- Ping-pong balls
- Rulers/meter sticks
- Calculators (optional)
- Stopwatches (1 per group, optional)
- Tower Power Rules Sheets
- Tower Power Evaluations

Engagement activity

Display some pictures of towers. Lead students in a discussion of the important characteristics of towers, relating tower structures to geometry and other mathematical concepts. Ask students to identify what types of designs make structures stronger. Instruct students to draw strong towers, paying special attention to the joints. As students share their tower designs, discuss that taller towers need more joints to hold them together. Have students explain why.

Exploration activity

Hint: Set up a store area for the straws, tape, marshmallows, toothpicks, and ping-pong balls prior to beginning the activity.

1. Organize students into groups. Have each group designate a timekeeper, recorder, and materials manager. Note: These 3 roles must be assigned; however, other roles may be employed to match group size. Emphasize the fact that **everyone is expected to participate** in the activity.

2. Distribute a Tower Power Rules Sheet and Tower Power Evaluation to each student. Distribute rulers/meter sticks, stop watches, and calculators to each group. Give students time to review the handouts and ask questions.

3. Explain the activity's **learning goals**: to complete the activity cooperatively, to practice critical thinking skills, and to use mathematical principles.

4. Do not allow students to purchase materials before the project starts. Remind them that your store has a no-return/no-refund policy.

5. Before groups begin the task, remind them to focus on the evaluation criteria and make good use of the planning time by discussing questions like:

   ? To what should you give more attention: height or costs?

   ? Can you effectively concentrate on both at the same time?

   ? How are you going to make sure your tower is strong enough (i.e., it remains standing)?

6. Keep track of each group's purchases and check their evaluation sheets to ensure that they recorded them accurately. A tally sheet for tracking store purchases can be found on p. 393.

7. Monitor the clock and call time if necessary. Verify each tower's height and have groups explain their design processes. Have groups exchange evaluation sheets and confirm each others' calculations. Then, declare a winner.

Explanation

Solutions will vary. If necessary, a cost factor (cost per cm or cost per in) can be used to evaluate final scores.

Evaluation

Observe groups as they work, noting cooperation, participation, measuring skills, ability to follow rules and directions, problem-solving skills, and knowledge of mathematical principles.

Journal assignment
Have students complete these sentences in their journals:

- Design details that must be considered when building a tall structure include __________.
- Working in a cooperative group on this activity was ______________.
- If I could do this activity again, I would change ________________.

Extension activities

- Give groups an opportunity to discuss modifications to their tower designs, then let them build another tower that incorporates the changes.
- Compare the height of each group's structure to the heights of some well-known towers (e.g., the Eiffel Tower, the Leaning Tower of Pisa). Ask groups to calculate how many of their towers it would take to equal the heights of these famous towers.
- Allow students to choose building materials, set their costs, and determine building criteria. Then, instruct the groups to repeat the exploration activity with the new specifications.

Connections to other subjects

Art. Take students on a tour of your city or a large city nearby to study the architectural designs of the city's taller structures.

History. Instruct students to research the history of tall structures to discover when the first tower was built and for what specific purpose.

Science. Have students study the science of structures and their materials, including physics principles.

Language Arts. Have students read books about architectural designs and structures.

Home connections

Ask family members to talk with students about the benefits of cooperative work in the family and on the job.

Resources for teachers

Geometry in Architecture by William Blackwell, A.I.A (Key Curriculum Press, 1984)
Building Toothpick Bridges by Jeanne Pollard (Dale Seymour Publications, 1985)
Structures by Bernie Zubrowski (Cuisenaire, 1993)
Articles about famous towers (e.g., Tower of Pisa, Tower of Babel, Naxos: the Pyrogoi Towers) can be found on the Internet.

Tower Power Rules Sheet

Objective: Your group must build the tallest tower that will support a ping-pong ball at the lowest cost. You will be scored in 3 areas—**tower height, cost of materials, and construction techniques**—according to the criteria on the Tower Power Evaluation. Therefore, carefully consider the point values assigned to each evaluation area during the planning process.

Time limits: You will have **20 minutes to plan** your design. During this planning time, you may use pencil, paper, rulers/meter sticks, and calculators only. You may want to sketch and write down your ideas. Planning papers will be collected. When planning time is over, you will have **20 minutes to construct** your tower. Time is critical, so monitor it and use it wisely!

Materials: Materials may be purchased during the **construction time only**. Costs in the price list reflect minimum purchase amounts, you must buy at least 2 of each item to receive the listed price. **All purchases are final.** No returns, exchanges, or refunds.

Price List

1 plastic drinking straw	*$100*
1 in of masking tape	*10*
1 marshmallow	*50*
2 toothpicks	*10*

RULES - RULES - RULES - RULES - RULES - RULES - RULES - RULES

1. You must build your tower as high as possible.
2. The tower must be able to support a ping-pong ball, mounted on top of the structure or within the top 25% of it.
3. The ping-pong ball may not be altered or damaged during or after construction.
4. The height of each structure will be verified at the end of the construction period. Towers will be measured from the base to the top.
5. The structure must be **free standing**. It may not be taped, glued, mounted, or in any other manner stuck to the table or floor.
6. Only the materials person may purchase from the store.
7. You **may** be asked to provide the cost factor (i.e., cost per cm or in) of your structure.

Tower Power Evaluation Sheet

Team members:

Timekeeper ______________________ Materials Manager ______________________

Recorder ______________________ Other ______________________

Criteria	Possible points	Earned points
Tower height *(circle a range)*		
1′0″ to 1′6″ (30–46 cm)	20	
1′6″ to 2′0″ (46–61 cm)	40	
2′0″ to 2′6″ (61–76 cm)	60	
2′6″ to 3′0″ (76–91 cm)	80	
Over 3′0″ (91 cm+)	100	
Exact height =		Height score =
Cost of materials *(circle a range)*		
Up to $500	50	
$501 to $700	40	
$701 to $900	30	
$901 to $1100	20	
$1100 to $1300	10	
Over $1300	0	
Exact cost =		Cost score =
Construction technique		
Durability	20	
Neatness	30	
		Construction score =
Total Score	**200**	**Total score =**

Comments:

Materials Tally Sheet

Group *(by name of materials person)*	Number of drinking straws *($100 each)*	Inches of masking tape *($10 per in)*	Number of marshmallows *($50 each)*	Number of toothpicks *($10 for 2)*	Total

Size-wise

About this learning activity

Students will estimate a wide variety of measurements, then make the necessary calculations to check their estimates.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III ○	V ○	VII X
II ○	IV ○	VI X	VIII ○

Note: Each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th-grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Measuring devices (i.e., graduated cylinders, beakers, measuring cups, stopwatches, measuring tapes, meter sticks, yard sticks, rulers, balance scales, spring or platform scales, bathroom scale, and gram weights)
- Cups of various sizes (e.g., film canister, drinking cup, juice bottle)
- Water (in large container)
- Paper towels
- Small puzzle
- Masking tape
- Variety of objects (e.g., nail, pencil, roll of masking tape)
- 2–3 textbooks (taped together)
- 2 pieces of cardboard
- Box
- Stations signs
- Size-wise Stations worksheets

Engagement activity

This lesson works well without an engagement activity.

Exploration activity

1. Set up stations before students enter the classroom. Use the Size-wise Stations worksheet on pp. 397–399 as a guide. Do not place the measuring devices at the stations.

2. Show students the stations you have set up and distribute the Size-wise Stations worksheets.

3. Instruct students to circulate from station to station, making **estimates only** and recording them on their worksheets. Tell students that all estimates must be written in **ink** (to hinder the temptation to change them later). *(Hint: Place measurement devices in stations **after** the estimating portion of the activity is complete.)*

4. When estimating is complete, give students a break while you set up the measurement devices needed at each station. Then, instruct students to rotate through the stations again, taking and recording the necessary measurements.

Note: You may wish to give students a choice of measuring devices at each station (e.g., meter stick, tape measure, and steel rule).

5. Facilitate a class discussion with questions like:

? What methods did you use to make your estimates?

? At which station was your estimate the closest to the actual measurement?

? At which station was your estimate the least accurate?

? Which measurement system is easiest to use? Why?

Explanation

This activity is designed to help students develop "measurement sense." They are given the opportunity to estimate size, length, capacity, time, and weight in both the U.S. standard and metric measurement systems.

Evaluation

- Review students' worksheets.
- Change the items at each station and have students repeat the activity for a grade.

Journal assignment
Have students describe the process they used to estimate a measurement.

Extension activity

☼ Set up a place in the classroom to display the *Estimate of the Week*. Have students enter their estimates for the displayed item each week and give a weekly prize for the closest estimate.

Home connections

Encourage students to bring in items from home that fit various measurement criteria. For example, an item with a length of 20 cm or a container that holds 100 liquid ml.

Size-wise Stations

Station 1: Liquid volume

1. Estimate the amount of liquid each cup will hold in both milliliters and ounces.

2. Use the graduated cylinder, beakers, and measuring cups to find the actual volumes.

	Estimate		Actual	
	ml	oz	ml	oz
Cup A				
Cup B				
Cup C				

Station 2: Time

1. Estimate how long in minutes and seconds it will take to put the puzzle together.

2. Use the stopwatch to measure the actual time it takes. Record it in minutes and seconds.

Estimate		Actual	
min	sec	min	sec

Station 3: Length

1. Estimate the length of the classroom in meters and yards.

2. Measure the actual length.

Estimate		Actual	
m	yd	m	yd

Station 4: Vertical jump

1. Holding a piece of masking tape, jump and stick the tape on the wall. You must jump from a **standing position**. (You may not take a running start.) Estimate the distance between the floor and the tape on the wall.

2. Estimate your **reach height** (i.e., the height your fingertips reach when you stand flat-footed and reach with 1 hand as high as possible). Then, subtract your estimated reach height from the distance estimate you made in Step 1. This is your **vertical jump** estimate. Record it in both inches and centimeters.

Estimate		Actual	
cm	in	cm	in

3. Measure the distance in Step 1 and your reach height, then calculate your actual vertical jump measurement (i.e., tape height – reach height = vertical jump). Calculate it in both inches and centimeters.

Station 5: Lightweight

1. Estimate the weight of each object in grams and ounces.

2. Measure each object with the balance scale (gram weights) and the spring or platform scale (ounces).

	Estimate		Actual	
	g	oz	g	oz
Nail				
Pencil				
Tape roll				

Station 6: Heavyweight

1. Estimate the weight of the textbooks in kilograms and pounds.

2. Measure the textbooks with a spring or bathroom scale in both kilograms and pounds

Estimate		Actual	
kl	lb	kl	lb

Station 7: Area

1. Estimate the area of each piece of cardboard in both square inches and square centimeters.

2. Measure the actual area, using a ruler and the formula for area (A = L x W).

	Estimate		Actual	
	sq cm	sq in	sq cm	sq in
Cardboard A				
Cardboard B				

Station 8: Volume

1. Estimate the volume of the box in both inches and centimeters.

2. Measure the actual volume of the box, using the formula: V = L x W x H.

Estimate		Actual	
cm	in	cm	in

Delicious Lunches

About this learning activity

Students will plan and present a week of school lunch menus that meet nutritional, budget, and customer requirements.

Process skills

- ○ Building models
- ○ Categorizing or classifying
- ○ Communicating
- ○ Comparing
- ○ Controlling variables
- ○ Experimenting
- ○ Hypothesizing
- ○ Inferring
- ○ Interpreting data
- ○ Measuring
- ○ Observing
- ○ Ordering
- ○ Predicting
- ○ Reasoning
- ○ Recognizing relationships
- ○ Recording

Ohio mathematics proficiency strand(s)

I ○	III X	V ○	VII X
II X	IV ○	VI ○	VIII X

Note: Each related strand is marked with an "X."

Ohio mathematics proficiency outcomes

4th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24 ○ 25

6th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16 ○ 17 ○ 18 ○ 19 ○ 20 ○ 21 ○ 22
○ 23 ○ 24

9th–grade outcomes

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10 ○ 11
○ 12 ○ 13 ○ 14 ○ 15 ○ 16

Refer to Appendix D, pp. 422–427, for a detailed listing of the proficiency outcomes.

Materials

- Grocery store advertisements
- Cookbooks
- Nutrition information (e.g., books, websites)
- Various package labels (optional, but helpful)
- National School Lunch Program Requirements handouts

Engagement activity

Pose a problem by asking students these questions:

- ? Do you like your school lunch or do you simply tolerate it?
- ? If you were given the opportunity to plan an entire week of school lunches, what would you do?

Help students brainstorm ideas for improving the school lunches (e.g., survey students) and list their ideas on the board.

Exploration activity

1. Organize students into teams of 3–4. Distribute a National School Lunch Program Requirements handout to each student.

2. Instruct teams to conduct research (including, but not limited to, surveys), then develop 1 week of school lunch menus. Explain that they cannot exceed a budget of $__________ per student. Note: Choose any dollar amount that is appropriate for your situation.

3. Explain that students must create a **plan, survey, 1-week menu, nutrition outline** (containing calories, fat grams, protein, and related information for each food item) and **budget**. Tell students that their menus must be eye-catching and contain prices for each meal. (You may wish to have students use computer graphics or poster board and art supplies.)

4. Distribute the nutritional information resources and encourage students to find additional information on their own.

5. Have teams share their finished documents with the class. Let each student vote for his/her favorite 1-week lunch menu and explain the choice.

Hint: You may wish to have teams draft an evaluation rubric as the first task of the project.

Evaluation

Evaluate teams' projects with the following criteria:

- Were the teams' calculations correct?
- Did teams provide choices?
- Did the students interpret nutrition resources properly?
- Were survey results reported in an easily understandable way (e.g., a graph)?
- Did the students exhibit good estimation skills?
- Were menus attractive and creative?

Journal assignment
Ask students to explain what they would change about their school lunches and why. (You may also wish to have them predict cost and nutritional consequences of their recommendations.)

Extension activities

- Have students present their projects to other classes in their grade or the entire school, and let classmates vote for their favorite menus.
- Instruct students to use the weekly budget costs to calculate yearly costs.

Home connections

Ask family members to give students the opportunity to plan a week of menus at home.

Resources for students

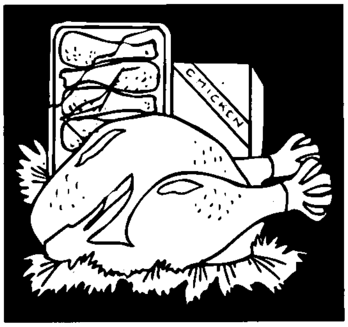
Nutrition-related websites:

- Food and Nutrition Information Center of the USDA at **www.nal.usda.gov/fnic/**; Healthy School Meals link
- Database of nutritional information on fast-food at **www.olen.com/food/**
- International Food Information Council (IFIC Foundation) provides food safety and nutrition information at **http://ificnfo.health.org/**
- Basal Metabolism Site calculates your basal caloric requirement based on age, height, and activity level; **www.room42.com/nutrition/basal.shtml**

National School Lunch Program Requirements

Meat/poultry/fish/bean
1 serving

Lunch meat
Hamburger
Hot dog
Tuna
Turkey
Peanut butter
Eggs
Peanuts



Bread/cereal
1 serving

Bread
Rolls
Cakes
Pasta
Rice
Tortillas
Bagels
Noodles
Pizza
Muffins

Fruit/vegetable
2 servings

Fresh fruits
Fruit juices
Salads
Corn
Potatoes
Carrots
Broccoli
Peppers

Milk
1 serving

Milk
Yogurt
Cheese
Butter
Ice cream
Cream cheese
Cottage cheese

Notes

Appendices

Notes

Appendix A: References

Adkinson, S., M. Fleer, eds. (1995). *Science With Reason.* London: Hodder and Stoughton Educational.

American Association for the Advancement of Science (1993). *Benchmarks for Science Literacy: A Project 2061 Report.* New York: Oxford University Press.

American Association for the Advancement of Science (1990). *Science for All Americans: A Project 2061 Report on Literacy Goals in Science, Mathematics, and Technology.* New York: Oxford University Press.

National Council of Teachers of Mathematics (1993). *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics (1991). *Professional Standards for Teaching Mathematics.* Reston, VA: NCTM.

Sadker, M. and D. Sadker (1994). *Failing at Fairness: How America's Schools Cheat Girls.* NY: Charles Scribner's Sons.

The Secretary's Commission on Achieving Necessary Skills (1992). *Learning a Living: A Blueprint for High Performance: A SCANS Report for America 2000.* Washington, DC: U.S. Department of Labor.

Appendix B: Resources

Many excellent books are suggested in the *Fun with Math* learning activities. In addition, the resources listed in this section can provide teachers and program coordinators with high-quality materials to enhance students' abilities to solve math-related problems.

Books

Anderson, C. and J. Haller (1995). *Brain Stretchers*. Pacific Grove, CA: Critical Thinking Press and Software.
Brain Stretchers includes exercises in visualization, mental and written computation, and deductive reasoning.

Brumbaugh, L. (1995). *Scratch Your Brain Where It Itches*. Pacific Grove, CA: Critical Thinking Press and Software.
This book helps stimulate mathematical reasoning and critical thinking through math games, tricks, puzzles, and brain teasers.

Dolan, D. and J. Williamson (1983). *Teaching Problem-Solving Strategies*. Menlo Park, CA: Addison Wesley Publishing Company.
This book includes a wide range of learning activities to help students systematically develop their mathematical problem-solving skills.

Downie, D. J. Stenmark, and T. Slesnick (1981). *Math for Girls and Other Problem Solvers*. Berkeley, CA: EQUALS, Lawrence Hall of Science.

Eisenhower National Clearinghouse (1995). *Active Learning with Hands-On Resources*. Columbus, OH: ENC.
This free publication describes and reviews high-quality, standards-based math and science resources. Copies can be obtained by calling 800/USA-LEARN or e-mailing info@enc.org.

Eisenhower National Clearinghouse (1998). *Ideas that Work: Mathematics Professional Development*. Columbus, OH: ENC.
This free publication includes 15 effective strategies for professional development. Strategies include immersion, curriculum development, pedagogical practice, and collaboration. Copies can be obtained by calling 800/USA-LEARN or e-mailing info@enc.org.

Eisenhower National Clearinghouse (1997). *Professional Development for Math and Science*. encFOCUS for Math and Science Education, Volume 4, Issue 4. Columbus, OH: ENC.
This free publication describes and reviews high-quality, standards-based math and science resources. Copies can be obtained by calling 800/USA-LEARN or e-mailing info@enc.org.

Erickson, T. (1989). *Get It Together: Math Problems for Groups, Grades 4–12*. Berkeley, CA: EQUALS, Lawrence Hall of Science.

Get It Together promotes cooperative problem solving among groups of 2–6 students. It includes 100 problems designed for teams using logic, geometry, algebra, measurement, probability, sequencing, and statistics, and suggests strategies for developing additional problems.

Erickson, T. (1989). *United We Solve: 116 Math Problems for Groups, Grades 4–6*. Berkeley, CA: EQUALS, Lawrence Hall of Science.

The problems in this book foster teamwork with 116 math challenges designed for groups. Problems involve patterns, spatial relationships, and proportion.

Fair, J. and M. Melvin (1986). *Kids Are Consumers, Too! Real-World Mathematics for Today's Classroom*. Menlo Park, CA: Addison-Wesley Publishing Company.

This book helps students in grades 3–8 learn mathematical skills including making and using charts and graphs, measuring, applying geometrical principles, earning and banking principles, shopping, and traveling. Everyday materials such as catalogs, advertisements, checkbook registers, maps, newspapers, timetables, and television schedules are used.

Fisher, L. (1982). *Super Problems*. Palo Alto, CA: Dale Seymour Publications.

Forsten, C. (1992). *Teaching Thinking and Problem Solving in Math: Strategies, Problems, and Activities*. NY: Scholastic Professional Books.

Fraser, D. (1980). *Newspaper Math*. Palo Alto, CA: Dale Seymour Publications.

Newspaper Math helps students solve practical, everyday arithmetic problems presented in the context of newspaper articles.

Gardella, F. (1995). *Problems Plus Series*. Austin, TX: Steck-Vaughn/Berrent Publications.

Developed for grades 1–8, this series is designed to assist in the development of students' thinking skills that are necessary for solving problems in and out of the classroom.

Hope, J., B. Reys, and R. Reys (1987). *Mental Math in the Primary Grades* and *Mental Math in the Middle Grades*. Palo Alto, CA: Dale Seymour Publications.

Each *Mental Math* book includes 36 lessons (and black-line masters) that systematically teach students to solve mathematical problems in their heads.

Illingworth, M. (1986). *Real-Life Math Problem Solving: 40 Exciting, Classroom-Tested Problems with Annotated Solutions*. NY: Scholastic Professional Books.

This book was developed for grades 4–8.

Johnson, D. R. (1986). *Every Minute Counts: Making Your Math Class Work and Making Minutes Count Even More.* Palo Alto, CA: Dale Seymour Publications.
This book gives high-quality, practical suggestions for managing the math classroom.

Kaseberg, A., N. Kreinberg, and Diane Downie (1990). *Use EQUALS to Promote Participation of Women in Mathematics.* Berkeley, CA: EQUALS, Lawrence Hall of Science.
This EQUALS publication helps educators provide equitable math problem-solving experiences for all students.

Krech, B. (1998). *Special Delivery: Putting Math to Work, Grades K–3.* Orangeburg, NY: Cuisenaire Company of America.
This book includes 2 Project Problems™ for each grade level. They are challenging, long-term (about five 40-minute class periods), hands-on, activity-based complex problems that provide students with opportunities to apply their math knowledge in situations where they will achieve real-life outcomes.

Lawrence Hall of Science. *Great Explorations in Math and Science (GEMS) Series.* Berkeley, CA: University of California.
Each GEMS guide comprises a comprehensive, ready-to-use unit on a given topic. Developed for use by non-math teachers, each involves students in hands-on, inquiry-based learning. Math titles are listed below. Other GEMS titles, including those pertaining to biology, ecology and physics, are also available. New titles are developed each year. GEMS materials are available through many suppliers, including William Sheridan Associates (listed in the Suppliers of Science Resources and Materials section of this appendix). GEMS staff can be reached directly at 510/642–7771; fax; 510/643–0309. Note: Topics and grade levels are listed after each title; however, most can be adapted for younger or older students.

Bubble Festival (bubble activities in learning station format, K–6)
Bubble-ology (bubble activities, 5–9)
Build It! Festival (construction, geometric challenges, and spatial visualization, K–6)
Group Solutions (cooperative logic activities, K–4)
Group Solutions, Too (cooperative logic activities, K–4)
In All Probability (probability and statistics, 3–6)
Height-O-Meters (triangulation, 6–10)
Math Around the World (multicultural math games, 5–8)
Shapes, Loops, and Images (logic and spatial relationships, all ages)
QUADICE (mental math, fractions, probability, 4–8)

Pedagogical handbooks in the GEMS series:
GEMS Teacher's Handbook
GEMS Leader's Handbook
Once Upon a GEMS Guide: Literature Connections
Insights and Outcomes: Assessment for GEMS Activities
A Parent's Guide to GEMS

Lee, M. and M. Miller (1997). *Real-Life Math Investigations: 30 Activities that Help Students Apply Mathematical Thinking to Real-Life Situations.* NY: Scholastic Professional Books.

Mokros, J., S. Russell, and K. Economopoulos (1995). *Beyond Arithmetic: Changing Mathematics in the Elementary Classroom.* Palo Alto, CA: Dale Seymour Publications.
Beyond Arithmetic provides teachers and administrators with practical suggestions for implementing national mathematics standards.

National Institute on Student Achievement, Curriculum, and Assessment (1997). *The Educational System in Japan: Case Study Findings.* Washington, DC: U.S. Department of Education.

Ohio Department of Education (1980). *Problem Solving...A Basic Mathematics Goal.* Books #1 (*Becoming a Better Problem Solver*) and #2 (*A Resource for Problem Solving*). Palo Alto, CA: Dale Seymour Publications.

Petreshene, S. *Brain Teasers!* Tuscon, AZ: Zephyr Press.
For use in grades 1–6, this book includes over 180 quick (5–15 minute) activities and worksheets that help students develop their thinking, reasoning, and memory skills.

Rowan, T. and B. Bourne (1994). *Thinking Like Mathematicians: Putting the K–4 NCTM Standards into Practice.* Portsmouth, NH: Heinmann Publishing.

Santi, T. (1998). *Math Ties: Problem Solving, Logic Teasers, and Math Puzzles—All "Tied" to the Math Curriculum.* Pacific Grove, CA: Critical Thinking Books and Software.
In addition to presenting over 25 learning activities, *Math Ties* includes suggestions for team-based problem solving.

Saunders, Hal. *When Are We Ever Gonna Have to Use This?* NY: Cuisenaire Publishing.
Designed for grades 7–12, this 34" x 22" poster includes over 350 work problems that use 60 different math topics arranged by occupation, math content, and three levels of difficulty. In addition, a 13-page book includes answers and a bibliography. Sold through Cuisenaire's secondary catalog and at www.cuisenaire.com.

Seymour, D. (1982). *Super Problems.* Palo Alto, CA: Dale Seymour Publications.

Skolnick, J. C. Langbort, and L. Day (1982). *How to Encourage Girls in Math and Science: Strategies for Parents and Educators.* Palo Alto, CA: Dale Seymour Publications.
Designed for grades K–8, this parent and educator guidebook examines effects of sex-role socialization on girls' skills, knowledge, and confidence in math and science, and traces the pattern of girls' involvement with math and science from early childhood through adolescence.

Souviney, R. (1981). *Solving Problems Kids Care About.* Glenview, IL: Good Year Books.
This book contains suggestions for teaching mathematical problem solving in grades K–8 and includes 34 reproducible black-line masters.

Souviney, R, M. Britt, S. Gargiulo, and P. Hughes (1992). *Mathematical Investigations: A Series of Situational Lessons.* Books 1–3. Palo Alto, CA: Dale Seymour Publications.
This series of books helps students make practical applications of mathematical principles on topics that include money matters, using scales, understanding finance, evaluating loans, taking a chance, and solving linear problems.

Stacey, K. and S. Groves (1985). *Strategies for Problem Solving: Lesson Plans for Developing Mathematical Thinking.* Palo Alto, CA: Dale Seymour Publishing.
This teacher resource for grades 7–10 provides sample lesson plans designed to help students use mathematical principles they have already learned to solve non-routine math problems.

Stenmark, J., V. Thompson, and R. Cossey (1986). *Family Math.* Berkeley, CA: Lawrence Hall of Science.
Family Math gives educators, parents, and students a wide variety of activities that target math and problem-solving skills.

Thompson, F. (1994). *Hands-On Math! Ready-to-Use Games and Activities for Grades 4–8.* West Nyak, NY: Center for Applied Research in Education.

Thompson, V. and K. Mayfield-Ingram (1999). *Family Math: The Middle School Years—Algebraic Reasoning and Number Sense.* Berkeley, CA: EQUALS.
This book gives parents tools for becoming advocates for their own children's achievement.

TOPS Learning Systems. Candy, OR: TOPS.
The TOPS Open-Ended Task Card and Structured Activity Sheet series provides thoughtfully sequenced activities for teaching and reinforcing specific math and science principles and process skills through hands-on, guided discovery. TOPS's non-profit status helps suppliers and TOPS keep the cost of these books at $8–$16 each. Most activities require low-cost, everyday materials. TOPS books can be purchased from many suppliers, including William Sheridan Associates (listed in the Suppliers of Science Resources and Materials section of this book), or directly from TOPS at 10970 Mulino Road, Canby, OR 97013; 888/722-9755; fax 503/266-5200; e-mail tops@canby.com.

Open-Ended Task Card Series: Designed for the grades noted beside each title, each series includes 16–36 lessons and teaching notes.

Graphing (grades 5–10)
Math Lab (grades 7–12)
Measuring Length (grades 5–10)
Metric Measure (grades 5–9 and 8–12)
Probability (grades 6–10)
Weighing (grades 5–10)

Structured Activity Sheet Series: Designed for grades 3–10, each book includes 20 ready-to-copy lessons. Skill levels are indicated beside each title.

Balancing (grades 3–7)
Metric Measuring (grades 5–9)
More Metrics (grades 6–10)

Vos, K. (ed.) (1996). *Monograph #1: A Perspective on Reform in Mathematics and Science Education.* National Council of Teachers of Mathematics for the Eisenhower National Clearinghouse for Mathematics and Science Education. Columbus, OH: ENC.

Washington, M. (1995). *Real Life Math Mysteries: A Kids' Answer to the Question, "What Will We Ever Use This For?"* Waco, TX: Prufrock Press.
Developed for students in elementary school, this book contains real-life math problems taken from a wide range of careers.

Additional Resources

Connect: A Magazine of Teachers' Innovations in K–12 Science and Math is published and distributed by Synergy Learning International, Inc. *Connect* is a valuable support for teaching problem solving skills through hands-on learning and interdisciplinary approaches. Each issue centers on a theme and includes strategies developed by fellow educators and support for helping students develop problem-solving skills. Subscriptions can be ordered by calling 800/769–6199 or by e-mailing connect@sover.net.

Learning Company Software:
The Geometric Golfer (grades 7–12)
Operation Neptune (grades 5–9)
The Secret Island of Dr. Quandry (grades 3–12)
Super Solvers Outnumbered! (grades 3–5)
TesselMania! (grades 3–12)

Math Trailblazers: A Mathematical Journey Using Science and Language Arts (1997). Dubuque, IA: Kendall/Hunt Publishing.
This series of kits is a research-based K–5 math program that encourages multiple solutions for problem solving and incorporates science and language arts. It is available for review at the Eisenhower National Clearinghouse for Mathematics and Science Education (described in the supplier section of this appendix).

*Mega Projects Cards**

Written for grades 1–6, this set of 50 cards, reproducibles, and 120-page teacher resource guide helps students develop real-life perspective by extending their classroom and community math explorations into the realms of earth and space science, biology, social studies, art, music, business, and sports.

*Techniques of Problem Solving (TOPS) Communication Cards**

This series helps students incorporate communication skills into solving thought-provoking math exercises. Each set includes 100 cards, an answer key, and suggestions for teachers. Three sets are available: grades 1–2, 3–4, and 5–6.

Sierra Software:

Lost Mind of Dr. Brain (grades 6–12)
Time Warp of Dr. Brain (grades 6–12)

Snyder, T. (1992). *Math Problem-Solving Courseware Series*. Watertown, MA: Tom Snyder Productions, Inc., sold by McGraw-Hill.

This series of software for grades 5–8 provides a method for teaching students problem solving strategies.

Vocational Marketing Services, *Introduction to Linear Measurement*.

This 30-minute videotape and accompanying student handbooks help students learn the English and Metric measurements systems in a lively, easy-to-understand manner. Conversions between systems and workplace applications are included. VMS can be reached at 800/343–6430.

* These card sets are available from suppliers of math materials, including William Sheridan & Associates and Summit Learning. Both are listed in the supplier section of this appendix.

Suppliers of Math and Science Resources and Materials

Center for Occupational Research and Development (CORD)

CORD develops and distributes applied math and applied science materials, as described in the Support Materials for Teachers and Parents section of this appendix. Contact CORD at P.O. Box 21206, Waco, TX 76702-1206; 800/231-3015.

Delta Education

Delta's hands-on math catalog for grades K-8 includes a wide variety of books and hands-on materials that can be helpful in teaching mathematical problem solving. Delta can be reached at 800/442-5444.

Eisenhower National Clearinghouse for Mathematics and Science Education (ENC)

The ENC was established to help K-12 teachers locate useful teaching materials. The Clearinghouse collects all types of materials at its repository in Columbus at The Ohio State University. The ENC reference desk is located at 1929 Kenny Road, Columbus, Ohio 43210-1079; 800/621-5785; 614/292-7784; fax 614/292-2066; e-mail info@enc.org.

Great Explorations in Math and Science (GEMS)

GEMS develops materials and provides in-service training to educators on topics related to inquiry-based science and math. Much of their work is done through regional support centers/network sites. The GEMS materials, found in Appendix B, can be obtained from William Sheridan and Associates (listed below) or directly from GEMS. GEMS staff can be reached at Lawrence Hall of Science, University of California, Berkeley, CA 94720-5200; 510/642-7771; fax 510/643-0309.

Summit Learning

Summit Learning's math manipulatives catalog includes books and hands-on materials helpful to teaching math problem solving. They can be reached at 800/777-8817.

Vocational Marketing Services

VMS publishes and supplies books, videos, and software for technology education (as well as many vocational, Tech Prep, and School-to-Work programs). They can be reached at 17600 South Williams Street, #6, Thornton, IL 60476; 800/343-6430.

William Sheridan & Associates

Sheridan & Associates provides a wide variety of science, math, and language arts materials. The company carries an amazing inventory of written resources as well—including most of those listed on the previous pages. They also create custom-designed learning kits on a wide variety of topics. Bill Sheridan and his staff have a reputation for friendly, personal service and locating almost anything a customer needs. They have several catalogs of their own, and supply the materials found in the Basic Science, Flinn Scientific, Connecticut Valley Biological catalogs, too. Their Discovery Store is open to the public, and located at 8311 Green Meadows Drive N., Lewis Center, Ohio 43035 (just north of Columbus). Call them at 800/433-6259; fax 740/548-0485.

Women's Educational Equity Act (WEEA) Program Publishing Center

WEEA distributes a wide variety of materials on topics related to educational equity, including disabilities, gender equity, careers, history, math, science, and technology. Materials and catalogs are available from WEEA at 55 Chapel Street, Suite 224, Newton, MA 02158–1060; 800/793–5076; e-mail WEEApub@EDC.org. Their web site can be found at http://www.edc.org/CEEC/WEEA.

Internet Sites*

Resource	Internet Address
Active Learning with Hands-On Resources (an ENC site)	http://enc.org/classroom/focus/ENC2868/2868.htm
Ask ERIC Lesson Plans	gopher://ericir.syr.edu:70/11/Lessons/Math
Catalog of Math Resources on WWW and the Internet	http://mthwww.uwc.edu/wwmahes/files/math01.htm
Clearinghouse for Mathematics and Science Education	http://www.nsta.org
Educational Computer Games (Math, Strategic Thinking)	http:/www.terc.edu/mathequity/gw/html.gwhome.html
Federal Resources for Educational Excellence	www.ed.gov/free
Improving Math & Science Education	http://www.learner.org/content/k12
Integrating Math and Science (an ENC site)	http://enc.org/classroom/focus/107941/7941.htm
Issues in Science and Technology	http://www.utdallas.edu/research/issues/
Lawrence Hall of Science	http://www.lhs.berkeley.edu/
MegaMath	http://www.c3.lanl.gov/mega-math/
NASA	http://quest.arc.nasa.gov/women/teachingtips.html
Professional Development for Math and Science (an ENC site)	http://enc.org/classroom/focus/pd/index.htm
U.S. Department of Energy	http://www.doe.gov
Using Children's Literature in Math and Science (an ENC site)	http://enc.org/classroom/focus/childlit/index.htm

*Internet addresses were correct upon *Fun with Math* publication. They are subject to change.

Appendix C: Math Vocabulary

Some students may need practice with words that are used in math books. To use this list as homework, have the students skim the list and read all familiar words. Check the ones that were known without hesitation. Then pick out five or six slightly harder words for each practice session. Remember to review recently learned words and to use praise routinely.

Basic Math Vocabulary

The following words are suggested for home vocabulary practice:

area

base
borrow
bushel

carry
chart
column
common
compare
complete
contain
corner
cube
cut

decimal
degree
difference
digit
divide
division

empty
endpoint
estimate
even
example

figures
fractions

gallon
graph
great
greatest
group

horizontal
hundred
hundreds

include

least
length

mark
mathematics
means
mile
million
minus
month
multiplication
multiply

near
nearer
nearest
ninth
numeral

odd
ones
ounces

peck
pint
plus
practice

quantity

rectangle
regroup
remainder
rename
Roman

section
set
solution
solve
square

temperature
tens
tenth
thermometer
thousand
times
ton
triangle
twice

unit

value
vertical
volume

weigh
weight
wide
width

yard
year

Advanced Math Vocabulary

The following words should be used for home practice when seen in lesson materials and recommended by a classroom or math teacher.

abacus
acre
algebra
angle
approximately
arc

billion

calculate
calculator
cardinal
census
centimeter
circumference
computer

cone
congruent
cubic
cylinder

denominator

equation

factor

geometry
gram

hundredths

identical
increment
index
infinity
integral
intersection

kilogram
kilometer

liter
logarithms

member
meter
metric
millimeter

negative
numerator

operation
ordinal

parallelogram
parallel
pattern
perimeter
perpendicular
polygon
population
positive

quotient

radius
ratio
root

segment
sphere
standard
subset
system

thousandths
trillion

variable

Definitions of Mathematical Terms

Area: The number of square units covering the interior of a closed plane figure.

Array: An orderly arrangement of objects in rows and columns. Rectangular arrays may be referred to as *n by m arrays*. The first number, n, indicates the number of rows; the second number, m, indicates the number of columns. Nonrectangular arrays also exist, such as Pascal's Triangle.

Attribute: A qualitative characteristic. Students are asked to group objects according to such attributes as color, size, shape, or other identifiable characteristics.

Congruent Figures: Geometric figures that are the same size and shape. All corresponding (matching) parts have the same measures.

Estimates: Reasonably close approximations of desired numerical results arrived at through various strategies such as front-end estimation, rounding, or successive approximations. Practicing estimation when measuring will help students develop a sense of the reasonableness of a measure and the proper units for the measure.

Experimental Probability: The numerical result of dividing the number of acceptable outcomes that occur within an experiment by the total number of outcomes generated during the experiment. As the number of events increases, the experimental probability (what actually happens) will approach the theoretical probability (what is expected to happen).

Face: The flat surface formed by polygons and their interiors.

Front-end Estimation: Rounding to the first, or front-end, digit to make an estimation.

Multiple Attributes: When objects have more than one characteristic (e.g., size, shape, or color).

Number Combinations: Using various groups of numbers and operations to generate a specified number. Number combinations help students develop flexibility with numbers.

Ordered Pairs: A pair of numbers, denoted x and y, is usually used to locate a point on a grid (rectangular coordinate system). The order is of utmost importance, as *2,3* is not the same as *3,2*. An ordered triple is used for 3-dimensional graphs.

Parallel Lines: Two lines that do not intersect, but lie in the same plane. For some applications, it is helpful to consider lines as being parallel to themselves.

Parallelogram: Any quadrilateral with two pairs of parallel sides. Rectangles and rhombuses are special cases of parallelograms.

Path: A path is a segment of a curve connecting two points. Sets of more than two points will give longer paths. Paths are generally used in discrete math with networks. Though most occur in two dimensions, paths can occur on other dimensional objects, such as the sides of a cube or a doughnut shape (called a torus).

Pattern Blocks: Manipulative sets consisting of triangles, squares, trapezoids, rhombuses, and hexagons that can be used to show relationships between geometric figures, fractional parts, and patterns.

Perimeter: The distance around any object. For a polygon, the perimeter is the sum of the measures of the sides. The circumference of a circle is analogous to perimeter. The measurement of perimeter is expressed in linear units.

Perpendicular Lines: Two lines, segments, or rays that intersect to form at least one right angle.

Pictograph: A graph that uses symbols to represent a quantity of objects or persons. The value of each symbol is shown in a key, which appears on the graph.

Polygon: A closed curve formed only by line segments that meet at their end points (called vertices).

Probability: A way of measuring what may happen in an uncertain situation. If there are "n" equally likely possibilities, of which one must occur and "s" is regarded as favorable, or as a success, then the probability of a success is "s/n." Students usually possess an intuitive sense of elementary probability, e.g., they know that the probability of flipping a head on a single toss of a fair coin is ½.

Rhombus: A parallelogram with four equal sides. A square is a special type of rhombus.

Right Angle: An angle whose measure is 90°.

Simple Closed Curves: A closed plane figure that does not intersect itself.

Symmetric Figure: Figures may have bilateral or rotational symmetry. A figure with bilateral symmetry can be folded on an imaginary line and match itself on each side of the line. A figure with rotational symmetry can be rotated to match itself, like a pentagon. Separate figures that can be flipped or rotated to coincide with each other are also called symmetric.

Trapezoid: A quadrilateral that has exactly one pair of parallel sides.

Triangle: A figure formed by three noncollinear segments called sides, intersecting at their end points. Also, a triangle is the figure formed by joining three noncollinear points.

Validate a Solution: The process of checking the correctness, reasonableness, and appropriateness of an answer to a problem situation.

Appendix D: Ohio Mathematics Proficiency Outcomes

Note: Additional information about Ohio Proficiency Outcomes, including practice tests, is available from the Ohio Department of Education's Assessment Center. Center staff can be reached at 65 S. Front Street, Room 207, Columbus, Ohio 43215-4183 and 614/466-0223.

Ohio 4th-Grade Mathematics Proficiency Outcomes

STRAND I – PATTERNS, RELATIONS, AND FUNCTIONS

1. Sort or identify objects on multiple attributes (e.g., size, shape, and shading).
2. Use patterns to make generalizations and predictions by:
   - determining a rule and identifying missing numbers in a sequence;
   - determining a rule and identifying missing numbers in a table of number pairs;
   - identifying missing elements in a pattern and justifying their inclusions; and
   - determining a rule and identifying missing numbers in a sequence of numbers or a table of number pairs related by a combination of addition, subtraction, multiplication, or division.

STRAND II – PROBLEM-SOLVING STRATEGIES

3. Select appropriate notation and methods for symbolizing a problem situation, translate real-life situations into conventional symbols of mathematics, and represent operations using models, conventional symbols, and words.
4. Identify needed information to solve a problem.
5. Explain or illustrate whether a solution is correct.

STRAND III – NUMBERS AND NUMBER RELATIONS

6. Decompose, combine, order, and compare numbers.
7. Illustrate or identify fractional parts of whole objects or sets of objects and like fractions greater than one, and add and subtract like fractions with illustrations and symbols.
8. Add, subtract, multiply, and divide whole numbers and explain, illustrate, or select thinking strategies for making computations.
9. Order fractions using symbols as well as the terms *at least* and *at most*.
10. Represent whole number value by:
    - Applying place value ideas.
    - Translating between words and symbols in naming whole numbers.
11. Add and subtract decimals.

STRAND IV – GEOMETRY

12. Apply congruence, symmetry, paths, simple closed curves, and the ideas of interior and exterior.
13. Recognize parallel, intersecting, and perpendicular lines, and right angles in geometric figures.
14. Determine properties of two-dimensional figures and compare shapes according to their characterizing properties, identify two-dimensional shapes on a picture of a three-dimensional object, and compare three-dimensional objects describing similarities and differences using appropriate standard or non-standard language.

STRAND V – ALGEBRA

15. Symbolize a keying sequence on a calculator and predict the display.
16. Model a problem situation using a number phrase/sentence and/or letters, understand the use of letters and symbols in statements such as $4b = 12$ or $3c = 15$ and find the value for a letter or symbol if the value for the other letter or symbol is given, and recognize the use of variables to generalize arithmetic statements applying the concept of odd and even numbers.

STRAND VI – MEASUREMENT

17. Apply the use of tools to measure lengths, using centimeters and inches, including recognizing the positions of whole numbers and fractions on a number line.
18. Apply the counting of collections of coins and bills (which could include one, five, and ten dollar bills) in a buying situation.
19. Illustrate the approximate size of units of length, capacity, and weight; choose an appropriate unit to measure lengths, capacities, and weights in U.S. standard and metric units; and relate the number of units that measure an object to the size of the unit as well as to the size of the object.
20. Determine perimeters and areas of simple straight line figures and regions without using formulas.
21. Use mental, paper-and-pencil, and physical strategies to determine time elapsed.

STRAND VII – ESTIMATION AND MENTAL COMPUTATION

22. Apply concept of place value in making estimates in addition and subtraction using front-end digits.
23. Round numbers and use multiples of ten to estimate sums, differences, and products and discuss whether estimates are greater than or less than an exact sum or difference.

STRAND VIII – DATA ANALYSIS AND PROBABILITY

24. Make or use a table to record and sort information (in a problem-solving setting using simple and complex patterns in nature, art, or poetry as setting) and make identifications, comparisons, and predictions from tables, picture graphs, bar graphs, and labeled picture maps.
25. Find simple experimental probabilities and identify events that are sure to happen, events sure not to happen, and those we cannot be sure about.

Ohio 6th-Grade Mathematics Proficiency Outcomes

STRAND I – PATTERNS, RELATIONS, AND FUNCTIONS

1. Apply the relation between doubling the side of a regular figure and the corresponding increase in area.
2. Determine the rule, identify missing numbers, and/or find the n^{th} term in a sequence of numbers or a table of numbers involving one operation or power.

STRAND II – PROBLEM-SOLVING STRATEGIES

3. Apply appropriate notations and methods for symbolizing the problem statement and solution process.
4. Identify needed and given information in a problem situation as well as irrelevant information.
5. Validate and/or generalize solutions and problem-solving strategies.

STRAND III – NUMBERS AND NUMBER RELATIONS

6. Compute with whole numbers, fractions, and decimals.
7. Find equivalent fractions.
8. Change freely between fractions and decimals.
9. Order combinations of whole numbers, fractions, and decimals by using the symbols $<$, $\leq$, $>$, $\geq$, and $=$ and/or by placing them on a number line.
10. Use ratios and proportions in a wide variety of applications.

STRAND IV – GEOMETRY

11. Visualize and show the results of rotation, translation, reflection, or stretching of geometric figures.
12. Recognize, classify, and/or use characteristics of lines and simple two-dimensional figures including circles; and apply models and properties to characterize and/or contrast different classes of figures including three-dimensional figures.

STRAND V – ALGEBRA

13. Use the distributive property in arithmetic computations.
14. Explain and reflect differences between calculators with arithmetic logic and calculators with algebraic logic when symbolizing a keying sequence and in the display as each key is pressed.
15. Use variables to describe arithmetic processes, to generalize arithmetic statements, and to generalize a problem situation.

STRAND VI – MEASUREMENT

16. Determine perimeters, areas, and volumes of common polygons, circles, and solids using counting techniques or formulas.
17. Convert, compare, and compute with common units of measure within the same measurement system.
18. Measure angles with a protractor.

STRAND VII – ESTIMATION AND MENTAL COMPUTATION

19. Apply appropriate strategies to find estimates of sums, differences, products, and quotients of whole numbers and determine whether the estimate is greater than or less than the exact result.
20. Estimate the sum, difference, product, or quotient of decimal numbers by rounding, and the sum, difference, or product of fractions and/or mixed numbers by rounding the fractions to 0, $1/2$, or 1.

STRAND VIII – DATA ANALYSIS AND PROBABILITY

21. Collect data, create a table, picture graph, bar graph, circle graph, or line graph and use them to solve application problems.
22. Read, interpret, and use tables, charts, maps, and graphs to identify patterns, note trends, and draw conclusions.
23. Apply the concept of average and calculate the arithmetic mean and mode of a given set of numbers.
24. Make predictions of outcomes of experiments based upon theoretical probabilities and explain actual outcomes.

Ohio 9th-Grade Mathematics Proficiency Outcomes

1. Compute with whole numbers, fractions, and decimals.
2. Compare, order, and determine equivalence of fractions, decimals, percents, whole numbers, and integers.
3. Solve and use proportions.
4. Round numbers to the nearest thousand, hundred, ten, one, tenth, and hundredth.
5. Solve problems and make applications involving percentages.
6. Select and compute with appropriate standard or metric units to measure length, area, volume, angles, weight, capacity, time, temperature, and money.
7. Convert, compare, and compute with common units of measure within the same measurement system.
8. Read the scale on a measurement device to the nearest mark and make interpolations where appropriate.
9. Recognize, classify, and use characteristics of lines and simple two-dimensional figures.
10. Find the perimeters (circumference) and areas of polygons (circles).
11. Find surface areas and volumes of rectangular solids.
12. Read, interpret, and use tables, charts, maps, and graphs to identify patterns, note trends, and draw conclusions.
13. Use elementary notions of probability.
14. Compute averages.
15. Solve simple number sentences and use formulas.
16. Evaluate algebraic expressions (simple substitutions).

Appendix E: Matrices of Learning Activities, Proficiency Outcomes, and Process Skills

Fun with Math for Grades 4–8

Ohio 4th-Grade Mathematics Proficiency Outcomes* for Each Learning Activity

Learning Activities ▽ / Outcomes ▷	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Strand I: Patterns, Relations, and Functions																									
Did Someone Say, "Patterns"?		✓																							
Patterns, Patterns		✓																							
Rubber Band Enlargements														✓											
Double Equals Quadruple																									
Double Exposure!																									
Will the Flowers Grow?																								✓	
Strand II: Problem-Solving Strategies																									
Get the Picture?			✓																						
Who's Afraid of Word Problems?			✓																						
Where's the Beef?				✓																					
What Do I Need?				✓																					
Robot Logic																									
Math Works in the Real World					✓																				
Strand III: Numbers and Number Relationships																									
Multiplying to Get Less									✓																
Paper-folding Magic									✓																
Comparing Fractions							✓		✓								✓								
Fun with Conversions																									
Scoring High with Fractions & Decimals									✓																
What's Your Order?									✓																
Equality—More or Less						✓			✓																
Sweet Ratios																									
Currency Conversions																									

* See pp. 423–424 for a description of each proficiency outcome.

Fun with Math for Grades 4–8
Ohio 4th-Grade Mathematics Proficiency Outcomes for Each Learning Activity

Learning Activities / Outcomes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Strand IV: Geometry and Spatial Sense																									
Which Way Did They Go?													✓												
Curves Ahead												✓	✓												
Lines and Dimensions														✓											
Fun with Reflections, Translations, Rotations, and Stretches												✓													
Screen Saver			✓									✓	✓	✓											
Strand V: Algebra																									
Distribution Breakdown																									
More Distribution																									
One Check or Two?																		✓							
It's In the Bag																✓									
Paper Equations																									
Strand VI: Measurement																									
Go the Distance!																	✓		✓	✓					
All Boxed In																				✓					
Who Wants Pi?																									
Box Builders																	✓		✓						
Measuring Around																									
Measuring in Metric																	✓		✓						
"Anglers" and Their Protractors																									
Strand VII: Estimation and Mental Computation																									
Front-end Estimation				✓	✓	✓					✓											✓	✓		
Estimating Areas and Volumes																	✓		✓		✓				
Rounding Decimals				✓	✓	✓																	✓		
Estimating Fractions and Decimals																	✓								

Fun with Math for Grades 4–8

Ohio 4th-Grade Mathematics Proficiency Outcomes for Each Learning Activity

Learning Activities / Outcomes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Strand VIII: Data Analysis and Probability																									
What Are the Chances?																								✓	✓
Do You Have Any Hearts?																									
Card Predictors																									
Probability Trees																									✓
Road Racers							✓	✓																✓	✓
Average M&Ms																									
Range, Mean, Median, and Mode				✓	✓	✓		✓																✓	
Graphing the Weather			✓		✓	✓		✓																	
Water Graph																								✓	
Multi-strand Activities																									
Clock-wise			✓	✓	✓		✓						✓	✓											
Snack Pack Analysis			✓	✓													✓							✓	
Volume Validity			✓											✓			✓		✓					✓	
The Giant Mystery			✓	✓	✓									✓			✓		✓				✓	✓	
Speedy		✓	✓	✓														✓						✓	
Tower Power			✓	✓	✓			✓						✓			✓	✓			✓			✓	
Size-wise							✓										✓		✓	✓					
Delicious Lunches			✓	✓				✓			✓														

Fun with Math for Grades 4–8

Ohio 6th-Grade Mathematics Proficiency Outcomes*

for Each Learning Activity

Learning Activities ▽ / Outcomes ▷	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Strand I: Patterns, Relations, and Functions																								
Did Someone Say, "Patterns"?		✓																						
Patterns, Patterns		✓																						
Rubber Band Enlargements	✓															✓								
Double Equals Quadruple	✓											✓			✓	✓	✓							
Double Exposure!	✓	✓				✓				✓	✓				✓	✓	✓							
Will the Flowers Grow?	✓																							
Strand II: Problem-Solving Strategies																								
Get the Picture?			✓																		✓			
Who's Afraid of Word Problems?			✓																					
Where's the Beef?				✓																				
What Do I Need?				✓																				
Robot Logic			✓	✓	✓																			
Math Works in the Real World					✓																	✓		
Strand III: Numbers and Number Relationships																								
Multiplying to Get Less						✓															✓			
Paper-folding Magic							✓														✓			
Comparing Fractions							✓		✓												✓			
Fun with Conversions								✓																
Scoring High with Fractions & Decimals								✓																
What's Your Order?									✓															
Equality—More or Less									✓															
Sweet Ratios										✓														
Currency Conversions										✓														

* See pp. 425–426 for a description of each proficiency outcome.

Fun with Math for Grades 4–8

Ohio 6th-Grade Mathematics Proficiency Outcomes for Each Learning Activity

Learning Activities / Outcomes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Strand IV: Geometry and Spatial Sense																								
Which Way Did They Go?												✓												
Curves Ahead											✓													
Lines and Dimensions												✓												
Fun with Reflections, Translations, Rotations, and Stretches											✓													
Screen Saver			✓								✓	✓												
Strand V: Algebra																								
Distribution Breakdown													✓											
More Distribution													✓											
One Check or Two?						✓							✓											
It's In the Bag															✓									
Paper Equations															✓									
Strand VI: Measurement																								
Go the Distance!																✓								
All Boxed In														✓										
Who Wants Pi?																✓								
Box Builders																✓								
Measuring Around																	✓							
Measuring in Metric																✓	✓							
"Anglers" and Their Protractors																		✓						
Strand VII: Estimation and Mental Computation																								
Front-end Estimation					✓	✓			✓										✓					
Estimating Areas and Volumes																✓			✓					
Rounding Decimals			✓		✓	✓														✓				
Estimating Fractions and Decimals																				✓				

Fun with Math for Grades 4–8

Ohio 6th-Grade Mathematics Proficiency Outcomes for Each Learning Activity

Learning Activities / Outcomes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Strand VIII: Data Analysis and Probability																								
What Are the Chances?																								✓
Do You Have Any Hearts?																								✓
Card Predictors																								✓
Probability Trees			✓	✓	✓	✓															✓	✓		✓
Road Racers							✓			✓											✓	✓		✓
Average M&Ms																							✓	✓
Range, Mean, Median, and Mode		✓	✓		✓	✓			✓												✓	✓	✓	
Graphing the Weather			✓	✓	✓	✓															✓	✓		
Water Graph																					✓	✓		
Multi-strand Activities																								
Clock-wise			✓	✓	✓		✓					✓						✓						
Snack Pack Analysis			✓	✓	✓	✓											✓				✓	✓	✓	
Volume Validity					✓							✓				✓	✓					✓		
The Giant Mystery			✓	✓	✓					✓		✓					✓		✓				✓	
Speedy		✓	✓	✓	✓	✓									✓						✓			
Tower Power			✓	✓	✓	✓						✓					✓							
Size-wise																✓	✓		✓	✓				
Delicious Lunches				✓		✓				✓									✓	✓	✓			

Fun with Math for Grades 4–8

Ohio 9th-Grade Mathematics Proficiency Outcomes*

for Each Learning Activity

Learning Activities ▽ / Outcomes ▷	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Strand I: Patterns, Relations, and Functions																
Did Someone Say, "Patterns"?																
Patterns, Patterns												✓				
Rubber Band Enlargements											✓					
Double Equals Quadruple											✓					
Double Exposure!											✓					
Will the Flowers Grow?											✓					
Strand II: Problem-Solving Strategies																
Get the Picture?												✓				
Who's Afraid of Word Problems?															✓	
Where's the Beef?																
What Do I Need?																
Robot Logic																
Math Works in the Real World												✓				
Strand III: Numbers and Number Relationships																
Multiplying to Get Less	✓						✓									
Paper-folding Magic	✓	✓														
Comparing Fractions		✓														
Fun with Conversions	✓	✓														
Scoring High with Fractions & Decimals	✓	✓														
What's Your Order?		✓														
Equality—More or Less	✓	✓														
Sweet Ratios			✓													
Currency Conversions			✓													

* See p. 427 for a description of each proficiency outcome.

Fun with Math for Grades 4–8
Ohio 9th-Grade Mathematics Proficiency Outcomes for Each Learning Activity

Learning Activities / Outcomes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Strand IV: Geometry and Spatial Sense																
Which Way Did They Go?									✓							
Curves Ahead								✓				✓				
Lines and Dimensions									✓							
Fun with Reflections, Translations, Rotations, and Stretches									✓							
Screen Saver									✓							
Strand V: Algebra																
Distribution Breakdown	✓															
More Distribution	✓															
One Check or Two?	✓															
It's In the Bag															✓	
Paper Equations															✓	
Strand VI: Measurement																
Go the Distance!										✓	✓					
All Boxed In											✓					
Who Wants Pi?										✓						
Box Builders											✓					
Measuring Around							✓									
Measuring in Metric							✓	✓								
"Anglers" and Their Protractors						✓										
Strand VII: Estimation and Mental Computation																
Front-end Estimation	✓	✓		✓						✓	✓					
Estimating Areas and Volumes											✓					
Rounding Decimals	✓	✓		✓												
Estimating Fractions and Decimals	✓						✓	✓								

Fun with Math for Grades 4–8

Ohio 9th-Grade Mathematics Proficiency Outcomes for Each Learning Activity

Learning Activities ▽ / Outcomes ▷	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Strand VIII: Data Analysis and Probability																
What Are the Chances?													✓			
Do You Have Any Hearts?													✓			
Card Predictors													✓			
Probability Trees												✓	✓			
Road Racers		✓			✓							✓	✓			
Average M&Ms														✓		
Range, Mean, Median, and Mode	✓	✓							✓					✓		
Graphing the Weather	✓	✓										✓				
Water Graph												✓				
Multi-strand Activities																
Clock-wise						✓		✓	✓							
Snack Pack Analysis	✓					✓	✓	✓				✓		✓		
Volume Validity						✓	✓	✓			✓	✓				
The Giant Mystery	✓		✓	✓		✓	✓	✓		✓				✓		
Speedy	✓														✓	✓
Tower Power	✓					✓	✓	✓	✓							
Size-wise						✓		✓								
Delicious Lunches	✓		✓	✓								✓				

Fun with Math for Grades 4–8
Ohio Process Skills for Each Learning Activity

Learning Activities ▽ / Process Skills ▷	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Strand I: Patterns, Relations, and Functions																
Did Someone Say, "Patterns"?			✓	✓				✓			✓	✓		✓	✓	✓
Patterns, Patterns		✓	✓	✓		✓	✓				✓	✓		✓	✓	✓
Rubber Band Enlargements	✓		✓	✓		✓				✓	✓		✓	✓	✓	✓
Double Equals Quadruple			✓	✓					✓		✓		✓	✓	✓	✓
Double Exposure!	✓		✓	✓						✓				✓	✓	
Will the Flowers Grow?	✓		✓	✓				✓	✓	✓	✓			✓	✓	✓
Strand II: Problem-Solving Strategies																
Get the Picture?	✓						✓	✓	✓					✓	✓	
Who's Afraid of Word Problems?			✓	✓			✓			✓	✓	✓	✓	✓	✓	
Where's the Beef?			✓	✓				✓	✓		✓			✓		✓
What Do I Need?			✓		✓				✓		✓		✓	✓		✓
Robot Logic			✓		✓				✓		✓			✓		✓
Math Works in the Real World			✓	✓				✓	✓		✓		✓	✓		

Process Skills Key (Process skills are described on pp. 12–13.)

1 Building models
2 Categorizing or classifying
3 Communicating
4 Comparing
5 Controlling variables
6 Experimenting
7 Hypothesizing
8 Inferring
9 Interpreting data
10 Measuring
11 Observing
12 Ordering
13 Predicting
14 Reasoning
15 Recognizing relationships
16 Recording

Learning Activities ▽ / Process Skills ▷	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Strand III: Numbers and Number Relationships																
Multiplying to Get Less	✓		✓				✓	✓	✓	✓	✓		✓	✓	✓	✓
Paper-folding Magic	✓		✓	✓		✓	✓	✓	✓		✓		✓	✓	✓	✓
Comparing Fractions		✓		✓		✓						✓			✓	✓
Fun with Conversions	✓														✓	
Scoring High with Fractions & Decimals			✓	✓		✓	✓	✓	✓		✓		✓	✓	✓	✓
What's Your Order?				✓								✓			✓	
Equality—More or Less		✓	✓	✓		✓		✓	✓		✓	✓		✓	✓	✓
Sweet Ratios			✓								✓					✓
Currency Conversions			✓	✓				✓						✓		
Strand IV: Geometry and Spatial Sense																
Which Way Did They Go?	✓	✓	✓	✓	✓	✓					✓			✓	✓	
Curves Ahead	✓		✓	✓		✓					✓	✓		✓	✓	✓
Lines and Dimensions	✓	✓	✓	✓		✓		✓	✓		✓			✓	✓	✓
Fun with Reflections, Translations, Rotations, and Stretches	✓			✓	✓	✓					✓	✓	✓	✓	✓	
Screen Saver	✓	✓	✓							✓						
Strand V: Algebra																
Distribution Breakdown			✓	✓		✓			✓		✓	✓	✓	✓		✓
More Distribution	✓		✓	✓					✓		✓			✓	✓	✓
One Check or Two?			✓						✓					✓	✓	✓
It's In the Bag						✓							✓	✓		✓
Paper Equations	✓		✓		✓						✓			✓		
Strand VI: Measurement																
Go the Distance!	✓					✓		✓	✓	✓	✓		✓	✓	✓	✓
All Boxed In	✓	✓		✓		✓		✓		✓	✓			✓	✓	✓
Who Wants Pi?										✓	✓					✓
Box Builders	✓		✓	✓	✓	✓					✓			✓	✓	✓
Measuring Around			✓	✓				✓		✓	✓			✓	✓	
Measuring in Metric						✓				✓						✓
"Anglers" and Their Protractors				✓				✓		✓	✓			✓	✓	✓

Learning Activities ▽ / Process Skills ▷	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Strand VII: Estimation and Mental Computation																
Front-end Estimation	✓	✓	✓	✓			✓	✓			✓	✓	✓	✓	✓	✓
Estimating Areas and Volumes			✓			✓							✓	✓		✓
Rounding Decimals			✓	✓								✓		✓	✓	✓
Estimating Fractions and Decimals										✓						✓
Strand VIII: Data Analysis and Probability																
What Are the Chances?		✓	✓										✓	✓	✓	✓
Do You Have Any Hearts?		✓	✓	✓					✓		✓		✓	✓		✓
Card Predictors											✓		✓			✓
Probability Trees	✓	✓	✓	✓				✓				✓	✓	✓	✓	✓
Road Racers			✓	✓		✓		✓	✓		✓	✓	✓	✓		✓
Average M&Ms				✓					✓	✓	✓			✓		✓
Range, Mean, Median, and Mode	✓	✓	✓	✓				✓			✓	✓	✓	✓	✓	✓
Graphing the Weather		✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Water Graph				✓			✓		✓	✓	✓	✓	✓		✓	✓
Multi-strand Activities																
Clock-wise	✓			✓						✓	✓	✓		✓		
Snack Pack Analysis			✓	✓				✓	✓	✓	✓			✓	✓	✓
Volume Validity			✓	✓		✓			✓	✓	✓			✓	✓	✓
The Giant Mystery			✓	✓				✓	✓	✓			✓	✓	✓	✓
Speedy	✓		✓	✓					✓				✓	✓	✓	✓
Tower Power	✓		✓							✓				✓		✓
Size-wise							✓			✓	✓		✓			✓
Delicious Lunches		✓	✓	✓				✓	✓		✓			✓		✓

Fun with Math

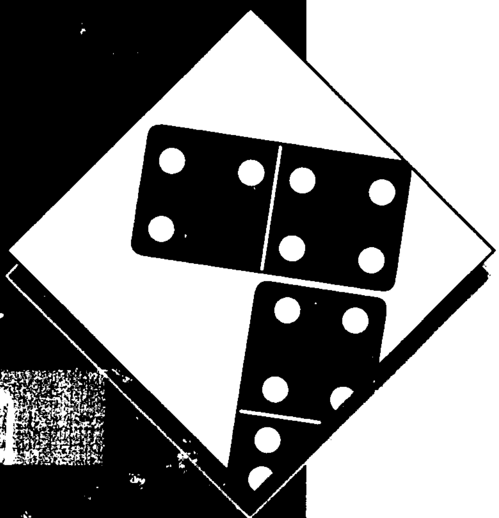
Real-Life Problem Solving

Additional Resources

This book series includes related science resources for teachers. *Fun with Physics: Real Life Problem Solving for Grades K–3* and *Fun with Physics: Real-Life Problem Solving for Grades 4–8* include a wide variety of exciting, inquiry-based learning activities in the areas of simple machines, electricity, liquids, and heat. Developed for non-science teachers, each learning activity includes step-by-step instructions for teachers, black-line masters, explanations of scientific principles, career connections, and more.

In-Service Training

The Vocational Instructional Materials Laboratory (VIML) at The Ohio State University provides coaching and training that will prepare teachers and trainers to effectively use the *Fun with Math* and *Fun with Physics* books and other VIML publications. Workshops on problem solving with math and physics for grades 9–12 and adults working in industry are also available. For further information about these services, contact the VIML at 800/848-4815 or 614/292-8300; 1900 Kenny Road, Columbus, Ohio 43210.



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